VICTORIA UNIVERSITY OF WELLINGTON TE HERENGA WAKA

4 June 2026 OIA 067

Kitmmy
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Tēnā koe Kitmmy

Thank you for your request for information under the Official Information Act 1982 (OIA) dated 22 May 2026 in which you requested:

> *"Please provide all test papers for the course EEEN415 for the years 2022 to 2025."*

## Response to request

Please find the following tests and exams for EEEN415 attached to this response:

- 2022 – Test 1;
- 2023 – Test 1;
- 2023 – Exam;
- 2024 – Test 1;
- 2024 – Exam;
- 2025 – Test 1; and
- 2025 – Exam.

Please refer to Annex A for 2022 – Test 2.

If you have any questions about this decision, you can contact us by email at: oiarequests@vuw.ac.nz. You have the right to seek an investigation and review by the Ombudsman of this decision by contacting www.ombudsman.parliament.nz or freephone 0800 802 602.

Nāku noa, nā

Kayana Shaw
Senior Advisor, Official Information and Privacy
Legal Services
**Te Herenga Waka—Victoria University of Wellington**

# Annex A – 2022 Test 2

You are given the task of placing actuators to control a new clothes modelling robot. The robot must be able to move smoothly between several set poses and remain in those poses indefinitely. Discuss the issues you would consider in choosing the actuator locations.

You have formed a discrete-time state space model for a new rocket motor using a combination of theory and experimentation. You now need to design an observer for the rocket, including sensor placement and observer pole placement. Discuss how you would approach this problem.

Both the minimum energy state transfer equation and one particular configuration of LQR are able to generate an optimal control signal u^\star(t) that will bring a system from one state to another. In contrast to these open loop control strategies, we often prefer to use closed loop strategies, where we measure the current state to produce a control signal in real time. Discuss the advantages and disadvantages of each broad type of control strategy.

The response of a discrete time system is found to be too sluggish, meaning that it takes too long for the system to return to its desired operating point after it has been disturbed. Discuss at least two factors that might limit how quickly the system responds and what might be done to address each issue?

The separation principle allows us to design regulators and observers separately using pole placement. The separation principle also holds when we use LQR or a Kalman filter. Consider the cases where we wish to combine

1. An LQR controller with a normal (Luenberger) observer, and
2. A conventional regulator with a Kalman filter.

Discuss anything that you might need to consider when designing systems using these combined approaches.

Consider a situation where you have a working Kalman filter with one or more sensors, each of which is returning data at one sample per second. You are given a new sensor that will sense some state (let's say x_1 for simplicity) once every ten seconds. Describe the various changes you would make to the Kalman filter implementation in order to add the new sensor.

# EEEN415 Advanced Control Systems
# Test One

7th September 2022

1. The test is open book.

2. The test is to be conducted under exam conditions. Communication is not permitted. Please remember that others will be taking the test after you, so don't discuss the test until you are sure that everyone has finished.

3. You should submit your answers as a **pdf** file using Blackboard. You can photograph and submit hand-written answers or if you prefer type them up. Hand writing the solutions has generally been preferred as easier and faster by previous groups of students.

4. You can submit figures via the online submission system. Figures should be in png or eps format and be in files called "fig1.png" or similar. You should refer to those figures in your written answers.

5. Code can be submitted, but it is unlikely to be read unless the results are wildly inconsistent with the written solution provided. **Do not** put critical information about your solutions in the .m files. Equally, do not spend time making your matlab code unnecessarily pretty.

6. Your answers should be written with conventional mathematical notation and terminology, not pseudo matlab. Where necessary refer to figure numbers in the text: "Figure 1 shows the response of the open loop system".

7. Submit your solutions via Blackboard. A link is available from the course web page. **Only** if that does not work should you email your solutions to christopher.hollitx@xxx.xxw.ac.nz with the subject "ECEN415-Test".

8. **Explain your reasoning.** Half marks will be awarded to solutions that work perfectly, but do not explain how the answers were obtained.

9. The test is designed to take an hour, but the system has been set to 90 minutes to ensure that you have plenty of time to submit your files.

1. A continuous time system is described by a state space model of

$$
\boldsymbol{A} = \begin{bmatrix} -2.8528 & -0.22911 & 0.010317 & 2.3604 & -0.2458 \\ 0.86101 & -3.1643 & 0.0098473 & 0.19876 & 0.70206 \\ -10.5019 & -1.1324 & -3.0004 & 43.4773 & -19.6592 \\ 0.43051 & -0.082139 & 0.0049237 & -2.9006 & 0.35103 \\ 0.011659 & 1.0056 & 0.0022897 & -1.5324 & -3.0819 \end{bmatrix}
$$

$$
\boldsymbol{B} = \begin{bmatrix} 0.80906 & 0 & 0 \\ 4.0963 & 1.118 & 0 \\ -69.8415 & 23.2809 & 23.2809 \\ 2.0482 & 0 & 0 \\ 3.7498 & 1.118 & -1.183 \end{bmatrix}
$$

$$
\boldsymbol{C} = \begin{bmatrix} 1.991 & 0 & 0 & 0.19003 & 0 \\ -0.86139 & -0.89443 & 0 & 3.5938 & 0 \\ -0.10441 & -0.42267 & 0.021477 & 0.86951 & 0.42267 \end{bmatrix}
$$

$$
\boldsymbol{D} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}
$$

$$
t \in \mathbb{R}_+
$$

A matlab file including this system definition is available on the course web site.

(a) [5 marks] What are the poles and corresponding modes of the open loop system?

(b) [5 marks] Design a regulator for the system that places a dominant pair of poles at $-5 \pm 5\text{j}$. Explain your choice of the placement of any additional closed loop poles.

(c) [5 marks] Design a regulator that instead places the poles in a Butterworth configuration. Ensure that the slowest closed loop mode decays at the same rate as the dominant pair in the previous controller.

(d) [5 marks] For each controller plot $\boldsymbol{y}$ and $\boldsymbol{x}$ for successive steps applied at the system inputs. That is, apply a step to the first input and wait for any transients to decay before applying a step at the second input (and so on).

(e) [5 marks] Demonstrate which of your two controller designs results in larger signals applied at $\boldsymbol{u}$. Explain your findings.

*Hint:* This question is not intended to be quantitative; a qualitative discussion of any pertinent aspects of system behaviour is all that is expected.

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END

# EEEN415 Advanced Control Systems
# Test One - 2023

$7^{\text{th}}$ of September

NAME:

Student Number:

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This test consists of FOUR questions. You should attempt all questions.

The test is out of a total of 50 marks.

The questions are worth 20, 10, 10 and 10 marks respectively.

The duration of the test will be 50 minutes.

OPEN BOOK

Calculators are permitted.

1. [20 marks]

A nonlinear, continuous time, system is found to have two equilibrium points. A linearisation around the first equilibrium at $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ has an eigenvalue of $\lambda_1 = 0.2$ associated with the eigenvector $\begin{bmatrix} \sqrt{2} \\ \sqrt{2} \end{bmatrix}$. The second eigenvalue for the linearisation is $\lambda_2 = -4$ with associated eigenvector $\begin{bmatrix} -\sqrt{2} \\ \sqrt{2} \end{bmatrix}$.

The second equilibrium point is at $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$ has a linearisation with $\lambda_1 = -10$ and the eigenvector $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ along with $\lambda_2 = -20$ associated with the eigenvector $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

(a) [8 marks] Sketch a phase portrait for the nonlinear system.



(b) [4 marks] on your phase portrait add the trajectory that you would expect the system state would trace out if it began at the initial conditions $\boldsymbol{x}(0) = \begin{bmatrix} 1.5 \\ -0.5 \end{bmatrix}$

(c) [8 marks] Consider the situation where the system is to be operated around the equilibrium point at $\boldsymbol{x} = \begin{bmatrix} 1 & 1 \end{bmatrix}^{\mathsf{T}}$. The system is placed into a new set of state variables $\boldsymbol{z}$ where $\boldsymbol{x} = \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix} \boldsymbol{z}$.

Sketch the phase portrait of the system linearised around this equilibrium point *in the new coordinate system*, including representative state trajectories.

2. [10 marks] A system has $\boldsymbol{\Phi}(t) = \begin{bmatrix} \exp(-2t) & 0 & 0 \\ 0 & \exp(-3t) & t\exp(-3t) \\ 0 & 0 & \exp(-3t) \end{bmatrix}$.

(a) [5 marks] If the system is at $\boldsymbol{x} = \begin{bmatrix} 2 & 0 & 1 \end{bmatrix}^{\mathsf{T}}$ at $t = 2$, where was it at at $t = 1$?

(b) [5 marks] Do you think that the system is a continuous time, or a discrete time system? Explain your reasoning.

3. [10 marks] A continuous time system with state governed by $\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u}$ has

$$\boldsymbol{B} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \\ 0 & -1 & -5 \end{bmatrix}.$$

(a) [7 marks] Give a choice for $\boldsymbol{u}$ that would cause $\dot{\boldsymbol{x}}_3 = 4$ units per second, while leaving the other state variables unchanged. You should ignore the action of the unknown $\boldsymbol{A}$ when answering this question.

(b) [3 marks] Would it be possible to construct other possibilities for $\boldsymbol{u}$ that also meet the goal? Explain why or why not.

4. [10 marks] Consider a system that cannot be diagonalised, but can be written in Jordan form, as follows.

$$\dot{\boldsymbol{x}} = \begin{bmatrix} -2 & 4 & 0 & 0 \\ -4 & -2 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & -1 \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \boldsymbol{u}$$

$$y = \begin{bmatrix} 1 & 0 & 1 & 0 \end{bmatrix} \boldsymbol{x}$$

An impulse is placed into each of the inputs in turn and the resulting dynamics allowed to die out before the application of the next impulse.

Sketch the response of *both* $\boldsymbol{x}$ and $\boldsymbol{y}$ in response to the impulses.

# EXAMINATIONS – 2023

# TRIMESTER 2

# FRONT PAGE

EEEN 415

**ADVANCED CONTROL SYSTEMS ENGINEERING**

01/11/2023

**Time allowed:** TWO HOURS

**Instructions:** **Answer all four questions**

Each Question is worth 30 marks

Open Book

Only silent non-programmable calculators or silent programmable calculators with their memories cleared are permitted in this examination.

1. **Dynamical Systems** **(30 marks)**

A nonlinear system is described in continuous time by the set of dynamical equations

$$\dot{x}_1 = x_1^3 - x_1 - x_2$$

$$\dot{x}_2 = - x_2$$

a) Find the equilibrium points for the system.

[3 MARKS]

b) Form a linearised model around EACH equilibrium point.

[4 MARKS]

c) What are the eigenvalues and eigenvectors associated with each of the equilibrium points? Categorise each as a stable or unstable node, a saddle, a centre or a stable or unstable focus.

[10 MARKS]

d) Sketch a phase portrait for the complete system, ensuring you show the directions of the eigenvectors (where possible). Ensure that you think about the direction of any spiralling or circulating structures.

[10 MARKS]

e) Explain whether the Hartman-Grobman theorem apply in the vicinity of each of your equilibrium points? Describe qualitatively why systems that do not satisfy this theorem cannot generally be treated by simple linearisation.

[3 MARKS]

**2. System Response** **(30 marks)**

A discrete time system is described by a state space model with

$$\boldsymbol{A} = \begin{pmatrix} 0.9 & 0 & 0 \\ 0 & 0.7 & 0.2 \\ 0 & -0.2 & 0.7 \end{pmatrix}, \boldsymbol{B} = \begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 0 & -1 \end{pmatrix}, \boldsymbol{C} = (-1 \quad 1 \quad 0) \text{ and } \boldsymbol{D} = (1 \quad 0).$$

a. Describe the modes that you would expect to see present in each of the state variables, and how they could be excited by the input signals.

[5 MARKS]

b. Imagine a series of steps were applied to each of the inputs in turn, with enough time left between each that the system dynamics had time to settle. Sketch the behaviour of each state variable, and the system output(s). Your diagram does not need to capture the discrete time nature of the response – just sketch smooth responses.

[15 MARKS]

c. A controllable system can be taken from an arbitrary state $\boldsymbol{x}(0)$ to another using an input signal given by

$$\boldsymbol{x}(\text{k}) = \Phi(\text{k})\boldsymbol{x}(0) + \text{M}_\text{c}\begin{pmatrix} \boldsymbol{u}(\text{k-1}) \\ \boldsymbol{u}(\text{k-2} \\ \vdots \\ \boldsymbol{u}(0) \end{pmatrix}$$

We could use this to build a regulator by setting $\boldsymbol{x}$(k)=0 and finding the $\boldsymbol{u}$ signal that would take us from our starting point at $\boldsymbol{x}$(0) back to zero. Discuss the advantages and disadvantages that such as scheme would have compared with a typical regulator.

[10 MARKS]

3. **Regulator Design** **(30 marks)**

A particular system that is driven by two actuators is described by a continuous time state space model

$$\dot{\boldsymbol{x}} = \begin{pmatrix} 0 & 1 \\ 16 & 0 \end{pmatrix} \boldsymbol{x} + \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} \boldsymbol{u}$$

$$y = \begin{pmatrix} 1 & 0 \end{pmatrix} \boldsymbol{x}$$

(a) Where are the open loop poles of the system?

[2 MARKS]

(b) Sketch and write expressions for the modes that you would expect to be present in the system's output.

[2 MARKS]

(c) Determine whether the system is controllable and/or stabilisable. Would the system be controllable and/or stabilisable if it were driven by each of its actuators independently?

[4 MARKS]

(d) You wish to design a regulator so that the closed loop system has a stable response that responds at least as quickly as $e^{-6t}$.

i. Discuss where you would place the poles of the closed loop system.

[2 MARKS]

ii. Use pole placement or any other suitable technique to show that there are multiple possible feedback matrices that could achieve your desired closed loop pole locations.

Note: you are **not** expected to find a solution for the feedback matrix in this case.

[10 MARKS]

iii. You decide to restrict the form of the feedback to $K = \begin{pmatrix} 0 & p \\ q & 0 \end{pmatrix}$ for some unknown p and q. Find p and q to produce the desired closed loop response.

[10 MARKS]

## 4. System Design **(30 marks)**

A system has $\boldsymbol{A} = \begin{pmatrix} -2 & 1 & 0 \\ 0 & -2 & 1 \\ 0 & 0 & -2 \end{pmatrix}$, $\boldsymbol{B} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{pmatrix}$ and $\boldsymbol{C} = (1 \quad 0 \quad 0)$, which leads to the controllability matrix $\mathrm{M_c} = \begin{pmatrix} 1 & 0 & -2 & 1 & 4 & -3 \\ 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 0 & 4 \end{pmatrix}$

and observability matrix $\mathrm{M_o} = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 4 & -4 & 1 \end{pmatrix}$.

a. Is the system controllable, stabilisable, observable and/or detectable?

[4 MARKS]

b. If it is controllable, in how many time steps would the reachability matrix become full rank? Alternatively, if it is not controllable, describe how you would determine whether it is stabilisable.

[2 MARKS]

c. If the system starts in state $\boldsymbol{x}(0) = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$, find some $\boldsymbol{u}$ that will take it to $\boldsymbol{x} = \begin{pmatrix} -3 \\ 1 \\ 4 \end{pmatrix}$ in a short a time as possible.

[12 MARKS]

d. Use Ackermann's equation, : $\boldsymbol{L} = \alpha(\boldsymbol{A})\mathrm{M_o^{-1}} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$, to find the feedback gain $\boldsymbol{L}$ that would place observer poles at s = -4. In this case the inverse of the observability matrix is

$$\mathrm{M_o^{-1}} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 4 & 4 & 1 \end{pmatrix}$$

[10 MARKS]

e. If you decided to use LQR to control the system, what would you consider when placing observer poles?

[2 MARKS]

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# EEEN415 Advanced Control Systems
# Test One - 2024

$2^{\text{nd}}$ of September

NAME:

Student Number:

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This test consists of FOUR questions. You should attempt all questions.

The test is out of a total of 50 marks.

The questions are worth 20, 10, 10 and 10 marks respectively.

The duration of the test will be 50 minutes.

OPEN BOOK

Calculators are permitted.

1. [20 marks]

A nonlinear, continuous time, system is found to have two equilibrium points. A linearisation around the first equilibrium at $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ has an eigenvalue of $\lambda_1 = 0.2$ associated with the eigenvector $\begin{bmatrix} \sqrt{2} \\ \sqrt{2} \end{bmatrix}$. The second eigenvalue for the linearisation is $\lambda_2 = -4$ with associated eigenvector $\begin{bmatrix} -\sqrt{2} \\ \sqrt{2} \end{bmatrix}$.

The second equilibrium point is at $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$ has a linearisation with $\lambda_1 = -10$ and the eigenvector $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ along with $\lambda_2 = -20$ associated with the eigenvector $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

(a) [8 marks] Sketch a phase portrait for the nonlinear system.



(b) [4 marks] on your phase portrait add the trajectory that you would expect the system state would trace out if it began at the initial conditions $\boldsymbol{x}(0) = \begin{bmatrix} 1.5 \\ -0.5 \end{bmatrix}$

(c) [8 marks] Consider the situation where the system is to be operated around the equilibrium point at $\boldsymbol{x} = \begin{bmatrix} 1 & 1 \end{bmatrix}^{\mathsf{T}}$. The system is placed into a new set of state variables $\boldsymbol{z}$ where $\boldsymbol{x} = \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix} \boldsymbol{z}$.

Sketch the phase portrait of the system linearised around this equilibrium point *in the new coordinate system*, including representative state trajectories.

2. [10 marks] An autonomous system has $\boldsymbol{\Phi}(t) = \begin{bmatrix} \exp(-2t) & 0 & 0 \\ 0 & \exp(-3t) & t\exp(-3t) \\ 0 & 0 & \exp(-3t) \end{bmatrix}$.

(a) [5 marks] If the system is at $\boldsymbol{x} = \begin{bmatrix} 2 & 0 & 1 \end{bmatrix}^{\mathsf{T}}$ at $t = 0$, where will it be at $t = 2$?

(b) [5 marks] Do you think that the system is a continuous time, or a discrete time system? Explain your reasoning.

3. [10 marks] A continuous time system with state governed by $\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u}$ has

$$\boldsymbol{A} = \begin{bmatrix} -4 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & -2 & 1 \\ 0 & 0 & 0 & -2 \end{bmatrix} \text{ and } \boldsymbol{B} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

(a) [5 marks] Is it possible to find a $\boldsymbol{u}_\text{o}$ that would hold the system at an equilibrium point $\boldsymbol{x}_\text{o} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}$? Note, that you don't need to find, $\boldsymbol{u}_\text{o}$ (though doing so would be one way of demonstrating that it is possible.)

*Hint:* $\boldsymbol{A}^{-1} = \begin{bmatrix} -\frac{1}{4} & 0 & 0 & 0 \\ 0 & -\frac{1}{3} & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & -\frac{1}{4} \\ 0 & 0 & 0 & -\frac{1}{2} \end{bmatrix}$

(b) [5 marks] Show that the system is controllable.

4. [10 marks] Consider a system that cannot be diagonalised, but can be written in Jordan form, as follows.

$$\dot{\boldsymbol{x}} = \begin{bmatrix} -2 & 4 & 0 & 0 \\ -4 & -2 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & -1 \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \boldsymbol{u}$$

$$y = \begin{bmatrix} 1 & 0 & 1 & 0 \end{bmatrix} \boldsymbol{x}$$

An impulse is placed into each of the inputs in turn and the resulting dynamics allowed to die out before the application of the next impulse.

Sketch the response of *both* $\boldsymbol{x}$ and $\boldsymbol{y}$ in response to the impulses.

# EXAMINATIONS – 2024
# TRIMESTER 2
# FRONT PAGE

EEEN415

**ADVANCED CONTROL SYSTEMS ENGINEERING**

01/11/2024

**Time allowed:** TWO HOURS

**Instructions:** **Answer all three questions.**

Each question is marked from a maximum of 40 marks

Closed Book

Only silent non-programmable calculators or silent programmable calculators with their memories cleared are permitted in this examination

A single sheet (double sided) of A4 paper with notes is permitted

**1**. A nonlinear, continuous time, sysem is described by the set of differential equations

$$\frac{dx_1}{dt} = f_1(\boldsymbol{x}) = (x_1 - 2x_2)x_1$$

$$\frac{dx_2}{dt} = f_2(\boldsymbol{x}) = (x_1 - 2)x_2$$

a) Find the equilibrium points of the system.

[5 MARKS]

b) Find a linear model around each of the equilibrium points.

[10 MARKS]

c) Find the eigenvalues at each equilibrium point and discuss the nature of the corresponding equilibrium point.

[10 MARKS]

d) Sketch the nonlinear phase portrait of the system. Be sure to calculate any useful eigenvectors and indicate them on your diagram.

[5 MARKS]

e) The system begins at the state $\boldsymbol{x}(0) = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$. Assuming that the evolution of the state can be well described by the linear model at the nearest equilibrium point, what would the state be three seconds later?

(If you were previously unable to find a linearised model, assume that $A = \begin{bmatrix} -2 & 0 \\ 0 & 3 \end{bmatrix}$ for this part of the question.)

[10 MARKS]

**2**. A system of three storage tanks has an approximate state space model given by

$$\dot{\boldsymbol{x}} = \begin{bmatrix} \mathbf{-2} & \varepsilon & \mathbf{0} \\ \mathbf{0} & \mathbf{-3} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{-8} \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} \boldsymbol{u} + \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} w \quad \text{and} \quad \boldsymbol{y} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \boldsymbol{x}$$

where ε is a (small) unknown constant that might be either negative or positive.

$x_1$ is the temperature in the first tank,

$x_2$ is the temperature in the second tank,

$x_3$ is the temperature in the third tank,

***u*** represents the operation of two heater/coolers connected to tanks two and three,

*w* represents stochastic flow of heat into/out of the tanks from the environment.

a) Calculate the controllability matrix of the system if only $u_1$ is used as a control input. Discuss the controllability and stabilisability of the system in this situation.

[5 MARKS]

b) Discuss the controllability and stabilisability of the system when both $u_1$ and $u_2$ are available for control.

[5 MARKS]

c) Explain how your answers to parts a) and b) would change if ε were to be zero.

[5 MARKS]

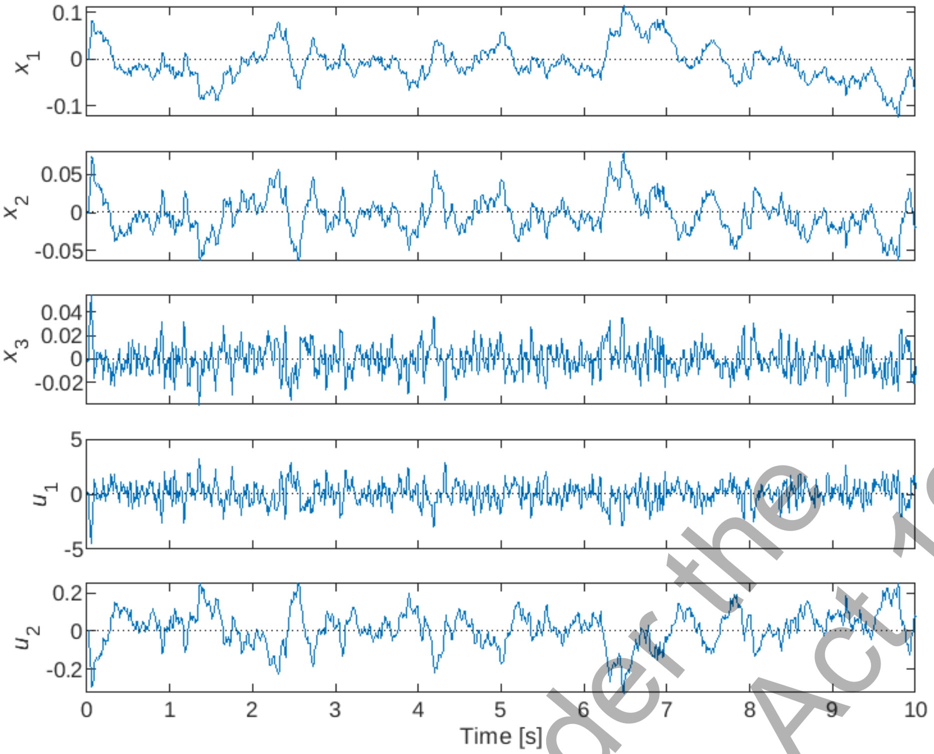


*Figure 1: Performance of an LQR controller*

d) An existing steady state LQR controller has been designed for the system using the cost matrices $Q = 1000\begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 81 \end{bmatrix}$, $R = \begin{bmatrix} 10 & 0 \\ 0 & 100 \end{bmatrix}$.

Recorded performance of the closed loop system is shown in figure 1. It is now desired to modify the controller so that the deviation of $x_2$ is reduced to approximately ±0.01 units, while $x_1$ is permitted to vary by no more than ±0.5 units.

Discuss changes that you would suggest to $Q$ and/or $R$ to bring about the necessary changes.

[5 MARKS]

e) You need to design a controller than ensures that the temperature of the third tank stays at the mean temperature of the other two tanks. Formulate an augmented model that you could use to build an integral controller that would achieve the desired outcome.

[15 MARKS]

f) If we require $x_3$ to be within 0.1 units of the average of the other two tanks, explain how you would modify the LQR cost matrices to incorporate the integral controller.

[5 MARKS]

**3**. A discrete time system is described by the state space model

$$x(t+1) = \begin{bmatrix} 0.9 & 1 & 0 \\ 0 & 0.9 & 0 \\ 0 & 0 & 0.8 \end{bmatrix} x(t) + w(t)$$

a) You can afford only a single sensor that can measure the difference between any two states.

Use the system's observability matrix to show that a sensor that measures $x_1 - x_3$ (or some other similar relation between $x_1$ and $x_3$) is the only option that makes the system observable.

[10 MARKS]

b) After how many time steps is the state fully observable?

[5 MARKS]

c) Use pole placement to design a dead beat observer for the system.

[10 MARKS]

d) In what ways would the response of a dead beat observer differ from that of a steady state Kalman filter?

[5 MARKS]

e) Imagine that a steady state Kalman filter was designed for this system and that the eigenvalues of the resulting closed loop system were slower than those that you designed in part c).

i. Imagine that each of the state variables were subjected to an impulsive disturbance in turn. For both your observer and the Kalman filter, sketch how the estimated state would then approach the true state.

[5 MARKS]

ii. Sketch how the uncertainty in the state estimate would evolve as a function of time for both your observer and the steady state Kalman filter when you first switch them on.

Hint: Sketch tr($P$) as a function of time. You can assume that the initial uncertainty is high.

[5 MARKS]

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# EEEN415 Advanced Control Systems
# Test One - 2025

$7^{th}$ of April

NAME:

Student Number:

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This test consists of FOUR questions. You should attempt all questions.

The test is out of a total of 70 marks.

The questions are worth 20, 20, 20 and 10 marks respectively.

The duration of the test will be 120 minutes.

A single A4 sheet of notes may be used.

Calculators are permitted.

1. A nonlinear, continuous time, system is found to have two equilibrium points. A linearisation around the first equilibrium at $\boldsymbol{x}_a^* = \begin{bmatrix}1\\1\end{bmatrix}$ has an eigenvalue of $\lambda_1 = 0.2$ associated with the eigenvector $\begin{bmatrix}1\\1\end{bmatrix}$. The second eigenvalue for the linearisation is $\lambda_2 = -4$ with associated eigenvector $\begin{bmatrix}-1\\1\end{bmatrix}$.

   The second equilibrium point is at $\boldsymbol{x}_b^* = \begin{bmatrix}-1\\-1\end{bmatrix}$ has a linearisation with $\lambda_1 = -10$ and the eigenvector $\begin{bmatrix}1\\0\end{bmatrix}$ along with $\lambda_2 = -20$ associated with the eigenvector $\begin{bmatrix}0\\1\end{bmatrix}$.

   (a) [8 marks] Sketch a phase portrait for the nonlinear system.

   (b) [4 marks] on your phase portrait add the trajectory that you would expect the system state would trace out if it began at the initial conditions $\boldsymbol{x}(0) = \begin{bmatrix}1.5\\-0.5\end{bmatrix}$

(c) [4 marks] Imagine that you wished to operate the system in the vicinity of the equilibrium point at $\boldsymbol{x} = \begin{bmatrix} 1 & 1 \end{bmatrix}^\mathsf{T}$. Explain how you would put the system into modal form.

(d) [4 marks] Sketch the phase portrait of the system linearised around this equilibrium point *in the new coordinate system*, including representative state trajectories.

2. (a) An autonomous system has $\boldsymbol{\Phi}(t) = \begin{bmatrix} \exp(-2t) & 0 & 0 \\ 0 & \exp(-2t) & 0 \\ 0 & 0 & \exp(3t) \end{bmatrix}$.

i. [4 marks] What is the $\boldsymbol{A}$ matrix for this system?

ii. [6 marks] At time $t = 1$ the system is at $\boldsymbol{x} = \begin{bmatrix} 2 & 4 & 8 \end{bmatrix}^\mathsf{T}$. What was the state at $t = -2$?

(b) An autonomous system has a state space model of

$$\dot{\boldsymbol{x}} = \begin{bmatrix} -2 & 1 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -1 \end{bmatrix} \boldsymbol{x}$$

i. [4 marks] What is the state transition matrix $\boldsymbol{\Phi}(t)$ for this system?

ii. [6 marks] If the system has at intial state $\boldsymbol{x}(0) = \begin{bmatrix} 2 & -3 & 4 \end{bmatrix}^{\mathsf{T}}$, at what time does $x_1 = 0$?

3. Sketch the phase portrait for each of the systems described below.

(a) [4 marks] The system has eigenvalues $\lambda_a = -2, \lambda_b = 10$, with associated eigenvectors being $\boldsymbol{v}_a = \begin{bmatrix}1 & 0\end{bmatrix}^\mathsf{T}$ and $\boldsymbol{v}_b = \begin{bmatrix}1 & 1\end{bmatrix}^\mathsf{T}$ respectively.

(b) [4 marks] The system has $\boldsymbol{A} = \begin{bmatrix}-20 & 16 \\ -32 & 12\end{bmatrix}$, with eigenvalues $\lambda_{a,b} = -4 \pm \mathrm{j}16$ and eigenvectors $\boldsymbol{v}_{a,b} = \begin{bmatrix}-1 \pm \mathrm{j} & -2\end{bmatrix}^\mathsf{T}$.

(c) [12 marks] The system is described by the set of differential equations

$$\dot{x}_1 = x_1^2 - 4$$
$$\dot{x}_2 = x_2$$

4. [10 marks] Consider a system that cannot be diagonalised, but can be written in Jordan form, as follows.

$$\dot{\boldsymbol{x}} = \begin{bmatrix} -2 & 4 & 0 & 0 \\ -4 & -2 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & -1 \end{bmatrix} \boldsymbol{x}$$

An impulse disturbance is placed onto each of the states in turn and the resulting dynamics allowed to die out before the application of the next impulse.

Sketch the response of each of the state variables in response to each of the impulses.

EXAMINATIONS – 2025

TRIMESTER 1

FRONT PAGE

**EEEN415**

**ADVANCED CONTROL SYSTEM ENGINEERING**

**13/06/2025**

**Time allowed:** TWO HOURS

**Instructions:** **Answer all four questions.**

Each question is marked from a maximum of 30 marks

Closed Book

Only silent non-programmable calculators or silent programmable calculators with their memories cleared are permitted in this examination

A single sheet (double sided) of A4 paper with notes is permitted

## 1. Regulator Design

A discrete time state space system has model

$$A = \begin{bmatrix} 0.7 & 0.2 \\ -0.2 & 0.7 \end{bmatrix},\ B = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix},\ C = \begin{bmatrix} 1 & 1 \end{bmatrix} \text{ and } D = 0.$$

a) What is the size of the *K* matrix needed to construct a state feedback regulator for this system?

[5 marks]

b) Use pole placement to design a regulator so that the poles of the closed loop system are at $s = 0.6 \pm 0.6j$.

> *Hint*: You may find that there are multiple possible solutions for *K*. If so, design any single regulator that will achieve the desired closed loop performance, and state any simplifying assumptions that you make.

[20 marks]

c) The system above uses two actuators for control. Discuss the advantages and disadvantages of removing either one of the actuators and controlling the system with the remaining actuator alone.

[5 marks]

## 2. Controllability

A state space system has

$$A = \begin{bmatrix} 0.8 & 1 & 0 & 0 \\ 0 & 0.9 & 0 & 0 \\ 0 & 0 & 0.8 & 1 \\ 0 & 0 & 0 & 0.6 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 \\ 0 & 1 \\ 0 & 0 \\ 1 & -1 \end{bmatrix}, C = [1 \quad 1 \quad 1 \quad -1] \text{ and } D = 0.$$

The resulting controllability matrix for the system is

$$\mathcal{M}_c = \begin{bmatrix} 0 & 1 & 0 & 1.8 & 0 & 2.34 & 0 & 2.68 \\ 0 & 1 & 0 & 0.9 & 0 & 0.81 & 0 & 0.73 \\ 0 & 0 & 1 & -1 & 1.4 & -1.4 & 1.48 & -1.48 \\ 1 & -1 & 0.6 & -0.6 & 0.36 & -0.36 & 0.22 & -0.22 \end{bmatrix}$$

a) Is the system controllable? If so, after how many time steps can the system be driven to zero from an arbitrary state? Ensure that you prove your answer mathematically.

[10 MARKS]

b) For the A matrix above we find that $(I - A)^{-1} = \begin{bmatrix} 5 & 50 & 0 & 0 \\ 0 & 10 & 0 & 0 \\ 0 & 0 & 5 & 12.5 \\ 0 & 0 & 0 & 2.5 \end{bmatrix}$.

Is it possible to regulate the system around an operating point of $x_o = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$?

If so, what is the required $u_o$ signal?

[10 MARKS]

c) Consider the continuous time state space model $A = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. Explain why the system is uncontrollable.

[5 MARKS]

d) When is a system stabilisable? If a system is uncontrollable, but stablisable, what does this mean for designing a practical regulator?

[5 MARKS]

### 3. Observer Design

A continuous time system is modelled with a state space system having

$$A = \begin{bmatrix} -2 & 1 \\ 0 & -2 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 2 & 2 \\ 1 & -1 \end{bmatrix} \text{ and } D = 0.$$

a) Is the system observable and/or detectable?

[5 MARKS]

b) Consider two modified versions of the system where only one of the system's existing sensors is used in each case. Are the two modified systems observable and/or detectable?

[5 MARKS]

c) Use Ackermann's equation to design an observer for the system that places the observer poles at $s = -10 \pm 10j$.

*Hint*: Ackermann's equation is $L = \chi_0(A)\mathcal{M}_o^{-1}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

[10 MARKS]

d) During use the sensor in your system breaks, such that its C matrix changes to $C = [1 \quad 0]$. Does the observer remain stable with your chosen $L$? Discuss whether you would expect the resulting observer to continue to provide an adequate estimate of the system state for practical use.

[10 MARKS]

**4. Short Explanation Questions.**

a) Explain the separation principle as it pertains to the design of a system containing both a regulator and an observer.

[5 MARKS]

b) Does the separation principle ensure that combining a Kalman filter with a regulator will work as expected? Explain why or why not.

[5 MARKS]

c) Consider a situation where an optimal (time varying) Kalman filter and a steady state Kalman filter both begin estimating the state of a system, with indentical large initial uncertainties. Sketch the evolution of the state uncertainty as a function of time for each filter.

[5 MARKS]

d) Explain the major drawback(s) of deadbeat controllers that results in them being rarely used in practice.

[5 MARKS]

e) Discuss whether using the deadbeat principle would be useful in designing an observer, including discussion of potential advantages and disadvantages.

[5 MARKS]

f) When using a Kalman filter to estimate the state of a system governed by $x = Ax + Bu + w$, we must calculate the prior estimate of state using

$$\hat{x}(t|t-1) = A\hat{x}(t-1|t-1) + Bu(t-1),$$

$$P(t|t-1) = AP(t-1|t-1)A^T + Q.$$

Explain why the process noise $w \sim \mathcal{N}(0, Q)$ does not appear in the prediction of *x*, but its covariance does appear in the prediction of the estimation covariance.

[5 MARKS]

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