



VICTORIA UNIVERSITY OF
WELLINGTON
TE HERENGA WAKA

CLASS TEST 2 – 2022

TRIMESTER 2

EEEN 220
SIGNALS, SYSTEMS
& STATISTICS 1

Time Allowed: FIFTY MINUTES

CLOSED BOOK

Permitted materials: Calculators. A single A4 sheet of formulae.

Instructions: Attempt ALL Questions.
NOT all questions have the same marks value.
There are 44 marks in total.

1. Conditional Probability

(10 marks)

Items are inspected for flaws by two quality inspectors. If a flaw is present, it will be detected by the first inspector with probability 0.9, and by the second inspector with probability 0.7. Assume the inspectors function independently.

- (a) If an item has a flaw, what is the probability that it is found by both inspectors? **(1 mark)**
- (b) If an item has a flaw, what is the probability that it is found by at least one inspector? **(1 mark)**
- (c) If the second inspector were to only examine those items that have been passed by the first inspector, what is the probability, for an item that has a flaw, that the second inspector will find it? **(1 mark)**

(d) If the probability that an item has a flaw is 0.1, and an item is passed by the first inspector, what is the probability that it actually has a flaw?

(3 marks)

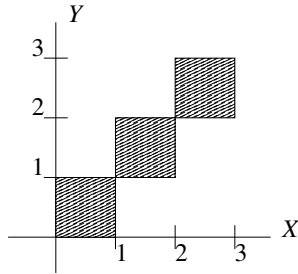
(e) If the probability that an item has a flaw is 0.1, and an item is passed by the both inspectors, what is the probability that it actually has a flaw?

(4 marks)

2. Joint Probability Density

(12 marks)

Consider the density $f_{X,Y}(x,y)$ shown below. The shaded area has a constant density, c . Elsewhere the density is zero. As always: explain all answers. Only providing answers results in no marks.



(a) Compute the value c of the density, where the density is not zero.

(1 mark)

(b) Determine the density $f_X(x)$ (a careful plot is ok).

(1 mark)

(c) Determine the function $f_{X,Y}(\frac{1}{2}, y)$ (a careful plot is ok).

(1 mark)

(d) Determine the density $f_{Y|X}(y|\frac{1}{2})$ (a careful plot is ok).

(2 marks)

(e) Determine $E\{X\}$

(2 marks)

(f) Determine the expectation $E\{XY\}$.

(3 marks)

(g) Compute the covariance between X and Y .

(2 marks)

3. Distributions

(6 marks)

Derive an expression for the mean and variance of a continuous random variable X which is uniformly distributed on the interval $[a, b]$.

(Note: There are *no* marks for the answers, only for the working).

(a) Mean:

(2 marks)

(b) Variance:

(4 marks)

4. Uncertainties

(5 marks)

We wish to measure a change between two independent measurements, expressed as a proportion of the initial value i.e., $y = (x_2 - x_1)/x_1$.

(a) If $x_1 = 1.11 \pm 0.01$ and $x_2 = 0.99 \pm 0.01$, what is the value of y ? (1 mark)

(b) What is the uncertainty in y ?

(4 marks)

5. Maximum Likelihood

(7 marks)

Derive the maximum likelihood estimator for the location parameter μ and the scale parameter b of the Laplace distribution (which has p.d.f $f_X(x) = \frac{1}{2b}e^{-\frac{|x-\mu|}{b}}$).

(Note: Most of the marks for this question are for careful explanation of what you are doing and the reasons for it).

6. Traffic

(4 marks)

A car insurance company is setting up a 0800 number and call centre for new customers. They estimate that during the busy hour the (Poisson) call arrival rate is 3 calls per minute. The mean call duration (assumed exponentially distributed) is 7 minutes. The company assumes that customers who encounter congestion on calling will immediately call their competitor, and don't want this to happen for any more than 5% of callers.

(a) What is the busy hour traffic load? (provide the value and the units)

(2 marks)

(b) How many operators does the call centre require to meet the availability target?

(2 marks)

Tables for EEEN220

Normal Distribution

$$\Pr[X \leq x] = \Phi(x) = 0.5 + \int_0^x \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dy$$

The table shows $\Phi(z) - 0.5$ as a function of $z > 0$.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998
3.6	.4998	.4998	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999
3.7	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999
3.8	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999
3.9	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000
4.0	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000

Erlang B Traffic Table

N	Pr[B]											
	0.0001	0.0005	0.001	0.005	0.01	0.02	0.05	0.1	0.15	0.2	0.30	0.40
1	.0001	.0005	.0010	.0050	.0101	.0204	.0526	.1111	.1765	.2500	.4286	.6667
2	.0142	.0321	.0458	.1054	.1526	.2235	.3813	.5954	.7962	1.000	1.449	2.000
3	.0868	.1517	.1938	.3490	.4555	.6022	.8994	1.271	1.603	1.930	2.633	3.480
4	.2347	.3624	.4393	.7012	.8694	1.092	1.525	2.045	2.501	2.945	3.891	5.021
5	.4520	.6486	.7621	1.132	1.361	1.657	2.219	2.881	3.454	4.010	5.189	6.596
6	.7282	.9957	1.146	1.622	1.909	2.276	2.960	3.758	4.445	5.109	6.514	8.191
7	1.054	1.392	1.579	2.158	2.501	2.935	3.738	4.666	5.461	6.230	7.856	9.800
8	1.422	1.830	2.051	2.730	3.128	3.627	4.543	5.597	6.498	7.369	9.213	11.42
9	1.826	2.302	2.558	3.333	3.783	4.345	5.370	6.546	7.551	8.522	10.58	13.05
10	2.260	2.803	3.092	3.961	4.461	5.084	6.216	7.511	8.616	9.685	11.95	14.68
11	2.722	3.329	3.651	4.610	5.160	5.842	7.076	8.487	9.691	10.86	13.33	16.31
12	3.207	3.878	4.231	5.279	5.876	6.615	7.950	9.474	10.78	12.04	14.72	17.95
13	3.713	4.447	4.831	5.964	6.607	7.402	8.835	10.47	11.87	13.22	16.11	19.60
14	4.239	5.032	5.446	6.663	7.352	8.200	9.730	11.47	12.97	14.41	17.50	21.24
15	4.781	5.634	6.077	7.376	8.108	9.010	10.63	12.48	14.07	15.61	18.90	22.89
16	5.339	6.250	6.722	8.100	8.875	9.828	11.54	13.50	15.18	16.81	20.30	24.54
17	5.911	6.878	7.378	8.834	9.652	10.66	12.46	14.52	16.29	18.01	21.70	26.19
18	6.496	7.519	8.046	9.578	10.44	11.49	13.39	15.55	17.41	19.22	23.10	27.84
19	7.093	8.170	8.724	10.33	11.23	12.33	14.32	16.58	18.53	20.42	24.51	29.50
20	7.701	8.831	9.412	11.09	12.03	13.18	15.25	17.61	19.65	21.64	25.92	31.15
21	8.319	9.501	10.11	11.86	12.84	14.04	16.19	18.65	20.77	22.85	27.33	32.81
22	8.946	10.18	10.81	12.64	13.65	14.90	17.13	19.69	21.90	24.06	28.74	34.46
23	9.583	10.87	11.52	13.42	14.47	15.76	18.08	20.74	23.03	25.28	30.15	36.12
24	10.23	11.56	12.24	14.20	15.30	16.63	19.03	21.78	24.16	26.50	31.56	37.78
25	10.88	12.26	12.97	15.00	16.13	17.51	19.99	22.83	25.30	27.72	32.97	39.44
26	11.54	12.97	13.70	15.80	16.96	18.38	20.94	23.89	26.43	28.94	34.39	41.10
27	12.21	13.69	14.44	16.60	17.80	19.27	21.90	24.94	27.57	30.16	35.80	42.76
28	12.88	14.41	15.18	17.41	18.64	20.15	22.87	26.00	28.71	31.39	37.21	44.41
29	13.56	15.13	15.93	18.22	19.49	21.04	23.83	27.05	29.85	32.61	38.63	46.07
30	14.25	15.86	16.68	19.03	20.34	21.93	24.80	28.11	31.00	33.84	40.05	47.74
40	21.37	23.41	24.44	27.38	29.01	31.00	34.60	38.79	42.48	46.15	54.24	64.35
50	28.87	31.29	32.51	35.98	37.90	40.26	44.53	49.56	54.03	58.51	68.46	80.99
60	36.62	39.40	40.80	44.76	46.95	49.64	54.57	60.40	65.63	70.90	82.70	97.63
70	44.58	47.68	49.24	53.66	56.11	59.13	64.67	71.29	77.26	83.32	96.95	114.3
80	52.69	56.10	57.81	62.67	65.36	68.69	74.82	82.20	88.91	95.75	111.2	130.9
90	60.92	64.63	66.48	71.76	74.68	78.31	85.01	93.15	100.6	108.2	125.5	147.6
100	69.27	73.25	75.24	80.91	84.06	87.97	95.24	104.1	112.3	120.6	139.7	164.3

Erlang C Traffic Table

N	Pr[W]											
	0.0001	0.0005	0.001	0.005	0.01	0.02	0.05	0.1	0.15	0.2	0.30	0.40
1	.0001	.0005	.0010	.0050	.0100	.0200	.0500	.1000	.1500	.2000	.3000	.4000
2	.0142	.0319	.0452	.1025	.1465	.2103	.3422	.5000	.6278	.7403	.9390	1.117
3	.0860	.1490	.1894	.3339	.4291	.5545	.7876	1.040	1.231	1.393	1.667	1.903
4	.2310	.3533	.4257	.6641	.8100	.9939	1.319	1.653	1.899	2.102	2.440	2.725
5	.4428	.6289	.7342	1.065	1.259	1.497	1.905	2.313	2.607	2.847	3.241	3.569
6	.7110	.9616	1.099	1.519	1.758	2.047	2.532	3.007	3.344	3.617	4.062	4.428
7	1.026	1.341	1.510	2.014	2.297	2.633	3.188	3.725	4.103	4.406	4.897	5.298
8	1.382	1.758	1.958	2.543	2.866	3.246	3.869	4.463	4.878	5.210	5.744	6.178
9	1.771	2.208	2.436	3.100	3.460	3.883	4.569	5.218	5.668	6.027	6.600	7.065
10	2.189	2.685	2.942	3.679	4.077	4.540	5.285	5.986	6.469	6.853	7.465	7.959
11	2.634	3.186	3.470	4.279	4.712	5.213	6.015	6.765	7.280	7.688	8.336	8.857
12	3.100	3.708	4.018	4.896	5.363	5.901	6.758	7.554	8.099	8.530	9.212	9.761
13	3.587	4.248	4.584	5.529	6.028	6.602	7.511	8.352	8.926	9.379	10.09	10.67
14	4.092	4.805	5.166	6.175	6.705	7.313	8.273	9.158	9.760	10.23	10.98	11.58
15	4.614	5.377	5.762	6.833	7.394	8.035	9.044	9.970	10.60	11.09	11.87	12.49
16	5.150	5.962	6.371	7.502	8.093	8.766	9.822	10.79	11.44	11.96	12.77	13.41
17	5.699	6.560	6.991	8.182	8.801	9.505	10.61	11.61	12.29	12.83	13.66	14.33
18	6.261	7.169	7.622	8.871	9.518	10.25	11.40	12.44	13.15	13.70	14.56	15.25
19	6.835	7.788	8.263	9.568	10.24	11.01	12.20	13.28	14.01	14.58	15.47	16.18
20	7.419	8.417	8.914	10.27	10.97	11.77	13.00	14.12	14.87	15.45	16.37	17.10
21	8.013	9.055	9.572	10.99	11.71	12.53	13.81	14.96	15.73	16.34	17.28	18.03
22	8.616	9.702	10.24	11.70	12.46	13.30	14.62	15.81	16.60	17.22	18.19	18.96
23	9.228	10.36	10.91	12.43	13.21	14.08	15.43	16.65	17.47	18.11	19.10	19.89
24	9.848	11.02	11.59	13.16	13.96	14.86	16.25	17.51	18.35	19.00	20.02	20.82
25	10.48	11.69	12.28	13.90	14.72	15.65	17.08	18.36	19.22	19.89	20.93	21.76
26	11.11	12.36	12.97	14.64	15.49	16.44	17.91	19.22	20.10	20.79	21.85	22.69
27	11.75	13.04	13.67	15.38	16.26	17.23	18.74	20.08	20.98	21.68	22.77	23.63
28	12.40	13.73	14.38	16.14	17.03	18.03	19.57	20.95	21.87	22.58	23.69	24.57
29	13.05	14.42	15.09	16.89	17.81	18.83	20.41	21.82	22.75	23.48	24.61	25.50
30	13.71	15.12	15.80	17.65	18.59	19.64	21.25	22.68	23.64	24.38	25.54	26.44
40	20.58	22.33	23.17	25.44	26.58	27.84	29.77	31.48	32.61	33.48	34.83	35.89
50	27.80	29.86	30.86	33.49	34.80	36.26	38.47	40.42	41.70	42.69	44.21	45.40
60	35.30	37.63	38.76	41.73	43.20	44.83	47.29	49.46	50.88	51.97	53.65	54.96
70	42.99	45.58	46.83	50.10	51.73	53.52	56.21	58.57	60.12	61.31	63.14	64.56
80	50.84	53.68	55.03	58.60	60.36	62.30	65.21	67.75	69.42	70.70	72.66	74.18
90	58.82	61.88	63.34	67.18	69.07	71.15	74.26	76.98	78.76	80.12	82.21	83.83
100	66.91	70.19	71.75	75.84	77.85	80.06	83.37	86.25	88.13	89.58	91.78	93.49

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