

**Assessment Schedule – 2023**

**Mathematics and Statistics: Apply algebraic procedures in solving problems (91027A)**

**Day 1**

Candidates must show algebraic working.

Equivalent methods of solving problems are accepted on condition that the candidate is demonstrating algebraic solutions/thinking at curriculum level 6.

Once a student has made an error, credit can be given from subsequent working which is at curriculum level 6.

Q	Expected Coverage	Grade (generated by correctly demonstrating the procedures listed in EN4) Requirements are for the student responses to be correct (ignoring numerical errors) unless the statement specifies consistent.
ONE (a)	$d = 2 \times (-3)^2 - 9 \times (2 \times -3 - 2) + 5$ $d = 2 \times 9 - 9 \times -8 + 5$ $d = 18 + 72 + 5$ $d = 95$	For award of u: • correct solution. Accept C.A.O.
(b)	$= \frac{(4x - 1)(x + 1)(x - 1)}{(x + 1)(x - 1)(4x - 1)}$ $= 1$	For award of u: ONE of: • numerator or denominator factorised • consistent simplification from their factorisation.
		For award of r: ONE of: • expression fully simplified • numerator and denominator both <b>expanded</b> correctly and then cancelled down to 1.

**Assessment Schedule – 2023****Mathematics and Statistics: Apply algebraic procedures in solving problems (91027B)****Day 2**

Candidates must show algebraic working.

Equivalent methods of solving problems are accepted on condition that the candidate is demonstrating algebraic solutions/thinking at curriculum level 6.

Once a student has made an error, credit can be given from subsequent working which is at curriculum level 6.

Q	Expected Coverage	Grade (generated by correctly demonstrating the procedures listed in EN4) Requirements are for the student responses to be correct (ignoring numerical errors) unless the statement specifies consistent.
ONE (a)	$42x^2 + 11x - 3 = (7x + 3)(6x - 1)$ i.e. $y = 6x - 1$ Allow C.A.O.  Accept $y = \frac{42x^2 + 11x - 3}{7x + 3}$	For award of u: <ul style="list-style-type: none"> <li>stating that <math>y = 6x - 1</math> or <math>y = \frac{42x^2 + 11x - 3}{7x + 3}</math></li> </ul>
(b)	$(8x + 2)(2x - 1) = (4x + 6)(4x - 3)$ $16x^2 - 8x + 4x - 2 = 16x^2 - 12x + 24x - 18$ $16x^2 - 4x - 2 = 16x^2 + 12x - 18 \quad \#1$ $-4x - 2 = 12x - 18$ $-2 + 18 = 12x + 4x$ $16 = 16x$ $x = 1$	For award of u: <ul style="list-style-type: none"> <li>expansion and simplification of a pair of brackets (LHS or RHS), forming a quadratic expression with three terms #1</li> </ul> For award of r: <ul style="list-style-type: none"> <li>equation solved to find <math>x = 1</math>.</li> </ul>
(c)	$\frac{3(3x + 2) - 4(4x - 1)}{12} \geq 2$ $\frac{9x + 6 - 16x + 4}{12} \geq 2$ $\frac{-7x + 10}{12} \geq 2 \quad \#1$ $-7x + 10 \geq 24$ $-7x \geq 14$ $x \leq \frac{14}{-7}$ $x \leq -2$ Accept $x \leq \frac{-14}{7}$ or $x \leq -2$ or $-2 \geq x$ as the final answer.	For award of u: ONE of: <ul style="list-style-type: none"> <li>correct arrangement for both numerator and denominator (does not need to be expanded or simplified). Accept <math>9x + 6 - 16x - 4</math> for the numerator</li> <li>consistent solution found (with either <math>\leq</math>, <math>\geq</math> or <math>=</math> sign).</li> </ul> For award of r: ONE of: <ul style="list-style-type: none"> <li>correct linear inequation at #1</li> <li>equation solved to find <math>x = -2</math></li> <li>inequation solved to find <math>x \geq -2</math></li> <li>consistently reverses inequality sign due to mult/div of a negative number.</li> </ul> For award of t: <ul style="list-style-type: none"> <li>inequation solved to find <math>x \leq -2</math>.</li> </ul>

<p>(d)</p>	$\frac{5g}{6} = \frac{h(g+4)}{5}$ $25g = 6h(g+4) \quad \#1$ $25g = 6hg + 24h$ $25g - 6hg = 24h \quad \#2$ $g(25 - 6h) = 24h \quad \#3$ $g = \frac{24h}{25 - 6h} \quad \#4$ <p>Accept other equivalent solutions.</p>	<p>For award of u: ONE of:</p> <ul style="list-style-type: none"> <li>• cross-multiply #1</li> <li>• consistently collecting terms involving <math>g</math> and terms not involving <math>g</math> on different sides of the equation #2</li> <li>• consistently factorising the pair of terms involving <math>g</math> #3</li> <li>• consistently rearranging by dividing by the bracket #4.</li> </ul> <p>For award of r: TWO of:</p> <ul style="list-style-type: none"> <li>• cross-multiply #1</li> <li>• consistently collecting terms involving <math>g</math> and terms not involving <math>g</math> on different sides of the equation #2</li> <li>• consistently factorising the pair of terms involving <math>g</math> #3</li> <li>• consistently rearranging by dividing by the bracket #4.</li> </ul> <p>OR</p> <ul style="list-style-type: none"> <li>• correctly states <math>h</math> in terms of <math>g</math>.</li> </ul> <p>For award of t: • correct rearrangement.</p>
<p>(e)</p>	$(4x - 1)(4x + 3) - x(x + 2) = 6$ $16x^2 + 12x - 4x - 3 - x^2 - 2x - 6 = 0$ $15x^2 + 6x - 9 = 0$ $5x^2 + 2x - 3 = 0$ $(5x - 3)(x + 1) = 0$ <p>Either <math>5x - 3 = 0</math></p> $x = \frac{3}{5}$ <p>OR</p> $x + 1 = 0$ $x = -1 \text{ Ignore as not appropriate.}$ <p>Units not required.</p>	<p>For award of u: ONE of:</p> <ul style="list-style-type: none"> <li>• forming correct expression <math>(4x - 1)(4x + 3)</math> simplified to <math>16x^2 + 8x - 3</math></li> <li>• forming correct equation for the shaded area</li> <li>• consistent simplification to a quadratic equation in three terms.</li> </ul> <p>For award of r: ONE of:</p> <ul style="list-style-type: none"> <li>• simplification to a quadratic equation in three terms</li> <li>• consistent solving of their quadratic equation, with evidence of negative value disregarded.</li> </ul> <p>For award of t: • correct positive solution found for the question, with evidence of negative value disregarded.</p>

Q	Expected Coverage	Grade (generated by correctly demonstrating the procedures listed in EN4) Requirements are for the student responses to be correct (ignoring numerical errors) unless the statement specifies consistent.
TWO (a)	$w = 3 \times (-3)^2 - 8 \times (3 \times -3 - 2) + 6$ $w = 3 \times 9 - 8 \times -11 + 6$ $w = 27 + 88 + 6$ $w = 121$	For award of u: <ul style="list-style-type: none"> <li>correct solution.</li> </ul> Accept C.A.O.
(b)	$= \frac{(5x+1)(x-2)(x+2)}{(x+2)(x-2)(5x+1)}$ $= 1$	For award of u: ONE of: <ul style="list-style-type: none"> <li>numerator or denominator factorised</li> <li>consistent simplification from their factorisation.</li> </ul> For award of r: ONE of: <ul style="list-style-type: none"> <li>expression fully simplified</li> <li>numerator and denominator both <b>expanded</b> correctly and then cancelled down to 1.</li> </ul>
(c)	$18x^2 + 24x = (3x + 4)^2$ $18x^2 + 24x = 9x^2 + 24x + 16$ $9x^2 = 16$ $x^2 = \frac{16}{9}$ $x = \pm \frac{4}{3}$ <p>Accept <math>x = \pm \sqrt{\frac{16}{9}}</math></p> OR Alternative method: $18x^2 + 24x = (3x + 4)^2$ $18x^2 + 24x = 9x^2 + 24x + 16$ $9x^2 - 16 = 0$ $(3x - 4)(3x + 4) = 0$ $x = \pm \frac{4}{3}$ <p>Allow answer in any form.</p>	For award of u: ONE of: <ul style="list-style-type: none"> <li>expansion and simplification of RHS</li> <li>consistently solves, giving both solutions</li> </ul> For award of r: <ul style="list-style-type: none"> <li>correctly solves for both solutions.</li> </ul>

(d)	<p>Let <math>F</math> be the first part of the journey and <math>S</math> be the second part of the journey.</p> $F + S = 1200$ $\Rightarrow S = 1200 - F$ $\frac{1}{4}F = \frac{1}{6}S$ $\Rightarrow 6F = 4S$ $6F = 4(1200 - F)$ $6F = 4800 - 4F$ $10F = 4800$ $F = 480$ <p>Then <math>S = 1200 - 480 = 720</math> km</p>	<p>For award of u:</p> <p>ONE of:</p> <ul style="list-style-type: none"> <li>forms the equation <math>\frac{1}{4}F = \frac{1}{6}S</math> or <math>6F = 4S</math></li> <li>consistent combining of their equations in one variable.</li> </ul>
	<p>OR</p> <p>Alternative Method:</p> <p>Let <math>x</math> be the distance covered in the first part of the journey; then <math>1200 - x</math> is the distance remaining.</p> $\frac{x}{4} = \frac{1200 - x}{6}$ $6x = 4(1200 - x)$ $6x = 4800 - 4x$ $10x = 4800$ $x = 480$ <p>Remaining distance is <math>1200 - 480 = 720</math> km</p> <p>Units not needed.</p> <p>Allow alternative algebraic methods.</p>	<p>For award of r:</p> <p>ONE of:</p> <ul style="list-style-type: none"> <li>combining of the equations in one variable</li> <li>consistent distances found for both parts of the journey.</li> </ul>
		<p>For award of t:</p> <ul style="list-style-type: none"> <li>correct distances found for both parts of the journey.</li> </ul>

<p>(e)</p> $\begin{aligned} & \left[ (2n+3)^2 - 3(2n+3) + 6 \right] - \left[ (2n+1)^2 - 3(2n+1) + 6 \right] \\ &= \left[ 4n^2 + 12n + 9 - 6n - 9 + 6 \right] - \left[ 4n^2 + 4n + 1 - 6n - 3 + 6 \right] \\ &= \left[ 4n^2 + 6n + 6 \right] - \left[ 4n^2 - 2n + 4 \right] \\ &= 8n + 2 \quad \#1 \\ &= 2(4n+1) \end{aligned}$ <p>This expression has a factor of 2, it is divisible by 2.</p> <p>OR</p> <p>Alternative method:</p> <p>Assume <math>n</math> is odd – not required to be stated.</p> $\begin{aligned} & \left[ (n+2)^2 - 3(n+2) + 6 \right] - \left[ n^2 - 3n + 6 \right] \\ &= \left[ n^2 + 4n + 4 - 3n - 6 + 6 \right] - \left[ n^2 - 3n + 6 \right] \\ &= \left[ n^2 + n + 4 \right] - \left[ n^2 - 3n + 6 \right] \\ &= 4n - 2 \\ &= 2(2n-1) \end{aligned}$ <p>This expression has a factor of 2, it is divisible by 2.</p> <p>OR</p> <p>Alternative method :</p> <p>Assume <math>n</math> is even – not required to be stated.</p> $\begin{aligned} & \left[ (n+3)^2 - 3(n+3) + 6 \right] - \left[ (n+1)^2 - 3(n+1) + 6 \right] \\ &= \left[ n^2 + 6n + 9 - 3n - 9 + 6 \right] - \left[ n^2 + 2n + 1 - 3n - 3 + 6 \right] \\ &= \left[ n^2 + 3n + 6 \right] - \left[ n^2 - n + 4 \right] \\ &= 4n + 2 \\ &= 2(2n+1) \end{aligned}$ <p>This expression has a factor of 2, it is divisible by 2.</p> <p>Accept any order of the differences considered.</p> <p>Allow alternative algebraic methods.</p> <p>OR</p> <p>Allow other algebraic methods used by considering other consecutive odd terms, e.g. <math>2n-1</math> and <math>2n+1</math>.</p>	<p>For award of u:</p> <p>ONE of:</p> <ul style="list-style-type: none"> <li>• correct expression for the difference between two successive odd-numbered patterns</li> <li>• consistent expansion and simplification of the square shaped brackets</li> <li>• consistently simplifies to an expression without any brackets (equivalent to #1)</li> <li>• Any valid numerical working (using one example) with a clear justification.</li> </ul>
	<p>For award of r:</p> <p>ONE of:</p> <ul style="list-style-type: none"> <li>• correctly simplifies to an expression without any brackets #1</li> <li>• consistently uses algebraic reasoning and explanation to justify clearly that the result is divisible by 2</li> <li>• Any valid numerical working (using two or more examples) with a clear justification.</li> </ul>
	<p>For award of t:</p> <ul style="list-style-type: none"> <li>• uses algebraic reasoning and explanation to justify clearly that the result is divisible by 2.</li> </ul>

Q	Expected Coverage	Grade (generated by correctly demonstrating the procedures listed in EN4) Requirements are for the student responses to be correct (ignoring numerical errors) unless the statement specifies consistent.
THREE (a)	<p>Perimeter = <math>8(3x + 2) = 88</math>  <math>24x + 16 = 88</math>  <math>24x = 72</math>  <math>x = \frac{72}{24} = 3</math></p> <p>OR</p> <p>Alternative method:  Perimeter = <math>8(3x + 2) = 88</math>  <math>3x + 2 = 11</math>  <math>3x = 9</math>  <math>x = \frac{9}{3} = 3</math></p> <p>Allow solution as an unsimplified fraction.</p>	<p>For award of u:</p> <ul style="list-style-type: none"> <li>correct solution for the value of <math>x</math></li> </ul> <p>Accept C.A.O.</p>
(b)	<p><math>\frac{4x}{4x-3} = \frac{x+6}{x+3}</math>  <math>4x(x+3) = (x+6)(4x-3)</math>  <math>4x^2 + 12x = 4x^2 - 3x + 24x - 18</math>  <math>12x = 21x - 18</math>  <math>18 = 9x</math>  <math>x = 2</math></p> <p>OR</p> <p>Alternative method:  <math>\frac{4x(x+3) - (x+6)(4x-3)}{(4x-3)(x+3)} = 0</math>  <math>\frac{4x^2 + 12x - (4x^2 + 21x - 18)}{(4x-3)(x+3)} = 0</math>  <math>4x^2 + 12x - 4x^2 - 21x + 18 = 0</math>  <math>-9x + 18 = 0</math>  <math>x = 2</math></p>	<p>For award of u:</p> <p>ONE of:</p> <ul style="list-style-type: none"> <li>correct arrangement for both numerator and denominator (does not need to be expanded or simplified). Accept <math>4x^2 + 12x - 4x^2 + 21x - 18</math> for the numerator</li> <li>consistent solution found.</li> </ul> <p>For award of r:</p> <ul style="list-style-type: none"> <li>correct value for <math>x</math> found.</li> </ul>

<p>(c)</p> $\frac{1}{2} \times (2x + 4)(x + 6) = 32$ $(x + 2)(x + 6) = 32$ $x^2 + 8x + 12 = 32$ $x^2 + 8x - 20 = 0$ $(x + 10)(x - 2) = 0$ <p>Either <math>x = -10</math> Ignore as not appropriate Or <math>x = 2</math></p> <p>OR</p> <p>Alternative method:</p> $\frac{1}{2} \times (2x + 4)(x + 6) = 32$ $(2x + 4)(x + 6) = 64$ $2x^2 + 12x + 4x + 24 - 64 = 0$ $2x^2 + 16x - 40 = 0$ $x^2 + 8x - 20 = 0$ $(x + 10)(x - 2) = 0$ <p>Either <math>x = -10</math> Ignore as not appropriate. Or <math>x = 2</math></p>	<p>For award of u: ONE of:</p> <ul style="list-style-type: none"> <li>• form and simplify a quadratic expression for the area of the triangle, with or without the 32 being used</li> <li>• consistent solution found, with evidence of the invalid value disregarded.</li> </ul>
	<p>For award of r:</p> <ul style="list-style-type: none"> <li>• correct solution found, with evidence of the invalid value disregarded.</li> </ul>



<p>(d)</p>	$2^{5x-2} \times (2^2)^{x+2} = (2^4)^x$ $2^{5x-2} \times 2^{2x+4} = 2^{4x}$ $2^{5x-2+2x+4} = 2^{4x} \quad \#1$ $2^{7x+2} = 2^{4x}$ $7x + 2 = 4x \quad \#2$ $3x = -2$ $x = -\frac{2}{3}$	<p>For award of u: ONE of:</p> <ul style="list-style-type: none"> <li>recognition of powers of 2 on both sides</li> <li>LHS or RHS correct at stage #1.</li> </ul> <p>For award of r: ONE of :</p> <ul style="list-style-type: none"> <li>forming the linear equation #2</li> <li>consistently forming an equation and solving for their <math>x</math>-value.</li> </ul> <p>For award of t: • correct solution found.</p>
<p>(e)</p>	<p>Let <math>T</math> be the number of 20 kg sacks. Let <math>F</math> be the number of 50 kg sacks.</p> $T + F = 60$ $20T + 50F = 1500$ $T + F = 60$ $2T + 5F = 150$ $2T + 2F = 120$ $2T + 5F = 150$ <p>Subtracting gives:</p> $3F = 30$ $F = 10 \text{ and } T = 50$ <p>Weight of 20 kg sacks will be <math>50 \times 20 = 1000</math> kg</p> <p>Weight of 50 kg sacks will be <math>10 \times 50 = 500</math> kg</p> <p>or alternative method:</p> $20(60 - F) + 50F = 1500$ $F = 10$ $T = 50$ <p>Candidate could use <math>x</math> and <math>y</math> as the variables. Allow alternative algebraic methods.</p>	<p>For award of u: ONE of:</p> <ul style="list-style-type: none"> <li>forms both equations</li> <li>consistent combining of their equations into one variable.</li> </ul> <p>For award of r: ONE of:</p> <ul style="list-style-type: none"> <li>consistent number of either 20 kg sacks or 50 kg sacks</li> <li>correct value for either 20 kg sacks or 50 kg sacks</li> <li>correct combining of the equations into one variable</li> <li>consistent value for both 20 kg sacks and 50 kg sacks.</li> </ul> <p>For award of t: ONE of:</p> <ul style="list-style-type: none"> <li>correct number of both 20 kg sacks and 50 kg sacks</li> <li>correct value for both 20 kg sacks and 50 kg sacks.</li> </ul>

**Assessment Schedule – 2023****Mathematics and Statistics: Investigate relationships between tables, equations and graphs (91028)****Evidence**

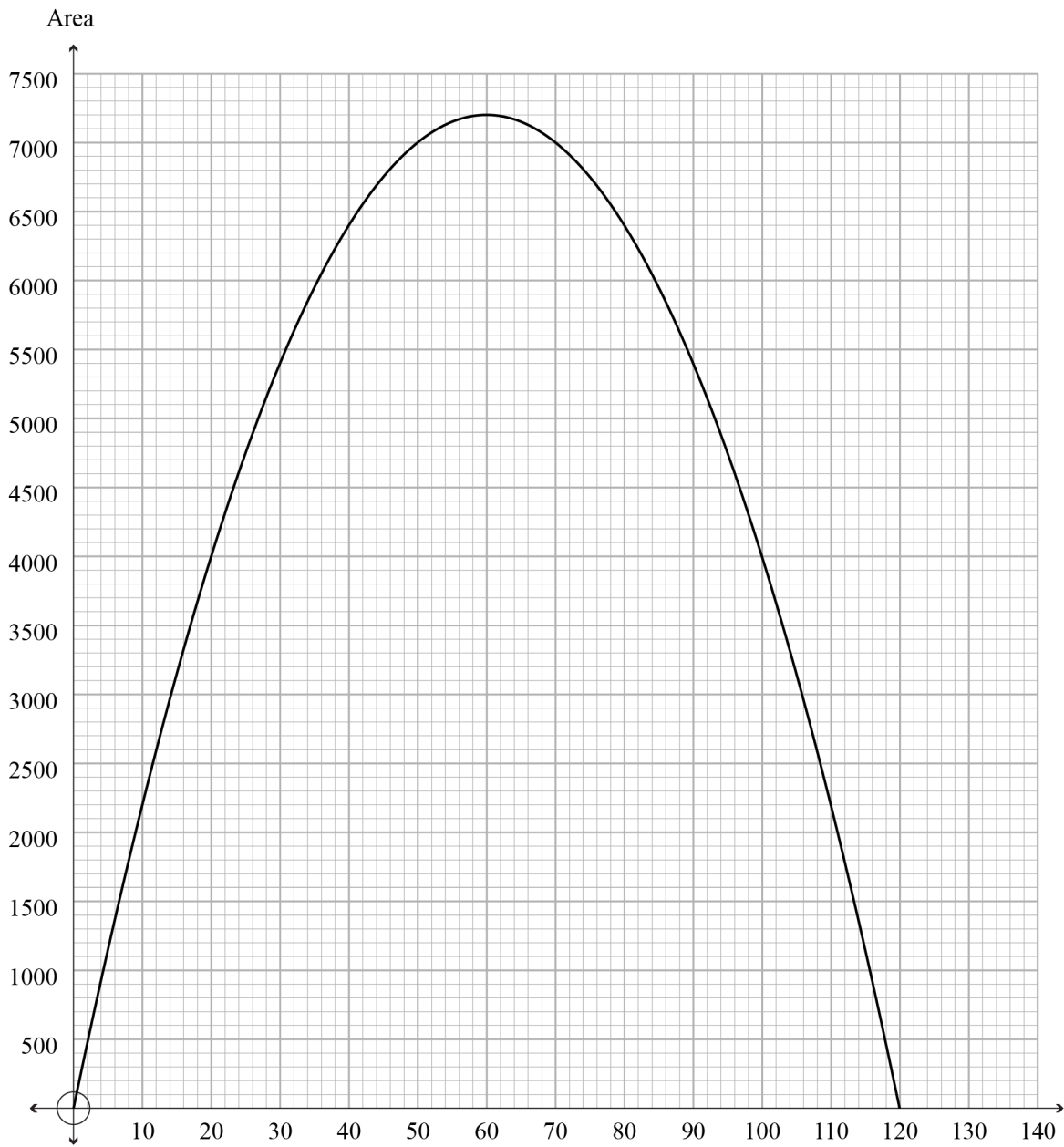
Q	Evidence	Achievement	Achievement with Merit	Achievement with Excellence
ONE (a)	$y = \frac{-5}{2}x + 15$ <p>Allow alternative forms. Allow C.A.O.</p>	<ul style="list-style-type: none"> <li>Correct equation.</li> </ul>		
(ii)	$y = \frac{5}{2}x + 15 + 10$ $y = \frac{5}{2}x + 25$ <p>Allow alternative forms. Allow C.A.O.</p>	<ul style="list-style-type: none"> <li>Correct equation after one of the transformations. OR Both transformations correct from wrong equation in (a)(i).</li> </ul>	<ul style="list-style-type: none"> <li>Correct equation after both of the transformations.</li> </ul>	
(b)(i)	$H = k(x - 60)^2 + 36$ <p><math>x = 0, y = 45</math> gives</p> $k = \frac{9}{3600} = \frac{1}{400} = 0.0025$ <p>i.e.</p> $H = \frac{1}{400}(x - 60)^2 + 36$ <p>OR Alternative formats:</p> $H = \frac{1}{400}x(x - 120) + 45$ <p>OR</p> $H = \frac{1}{400}x^2 - \frac{3}{10}x + 45$	<ul style="list-style-type: none"> <li>Equation given but with no <math>k</math>-value considered. OR Attempt made to find the value of <math>k</math> in a correct set up of the equation. OR C.A.O.</li> </ul>	<ul style="list-style-type: none"> <li>Correct equation for <math>H</math>, including full and clear working.</li> </ul>	
(ii)	<p>Possible changes are :</p> <p>The whole graph could be shifted downwards. This would represent shifting downwards where the chain fixes onto the post. This would be shown in the equation by reducing the size of the constant at the end.</p> <p>e.g. <math>H = \frac{1}{400}(x - 60)^2 + 20</math></p> <p>This would lower the chain totally by 16 cm. OR other examples where the chain is lowered.</p>	<ul style="list-style-type: none"> <li>Valid suggestion of how the equation should be changed. OR Example of equation of new design.</li> </ul>	<ul style="list-style-type: none"> <li>Valid suggestion of how the equation should be changed with an example equation AND Description of the minimum point of the chain fence, in context .</li> </ul>	

(c)	<p>Total perimeter (3 sides).  <math>2x + y = 240</math>  <math>y = 240 - 2x</math>                  Area = <math>x(240 - 2x)</math></p> <p>Allow other versions of this equation, e.g. <math>y = -2(x - 60)^2 + 7200</math>.                  (Allow any correct equation which starts with  <math>x + 2y = 240</math>)                  e.g. <math>y = -\frac{1}{2}(x - 120)^2 + 7200</math></p> <p>Table produced of the relationship between the two sides of the grassed space and their area with at least 5 correct values.</p> <p>Graph produced relating length of one side and area.</p> <p>Evidence of the use of tables, equations, and graphs to model the area of the grassed space as the lengths of the sides change.</p> <p>Sample comments:</p> <ul style="list-style-type: none"> <li>• Maximum area is 7200 m<sup>2</sup>.</li> <li>• Maximum area is when <math>x = 60</math> m and <math>y = 120</math> m.</li> <li>• Graph and area size is symmetrical.</li> <li>• Minimum area is 0 cm<sup>2</sup> (theoretically).</li> <li>• Rate of increase of the area changes for different <math>x</math>-values.</li> <li>• The graph will be a continuous one, as all different <math>x</math>-values are possible, if measurements are taken accurately.</li> <li>• In reality, some of the <math>x</math>-values close to 0 or close to 120 are likely to be inappropriate for the council to design their grassed area with these dimensions.</li> </ul>	<ul style="list-style-type: none"> <li>• Forming equation for area in terms of only one variable.</li> <li>OR</li> <li>Table only with one non-trivial comment.</li> <li>OR</li> <li>Graph only with ONE non-trivial comment.</li> <li>OR</li> <li>Finding maximum area only.</li> <li>OR</li> <li>Table and graph drawn with no comments.</li> </ul>	<ul style="list-style-type: none"> <li>• Evidence of only two aspects of tables, equations, and graphs.</li> <li>AND</li> <li>TWO non-trivial comments.</li> </ul>	<p><b>E7 / T1</b></p> <p>Evidence of table of values.                  AND                  Graph drawn.                  AND                  Formula for area provided.                  BUT                  Only maximum area discussed.                  OR                  As evidence for E8 but graph is discrete or of poor quality.</p> <p><b>E8 / T2</b></p> <p>Evidence of table of values.                  AND                  Graph drawn.                  AND                  Formula for area provided.                  AND                  At least three valid non-trivial comments.</p>
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N0	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE question attempted towards solution.	1u	2u	3u	2r	3r	1t	2t

### Question One

$x$	$y = 240 - 2x$	$A = x(240 - 2x)$
0	240	0
20	200	4000
40	160	6400
60	120	7200
80	80	6400
100	40	4000
120	0	0

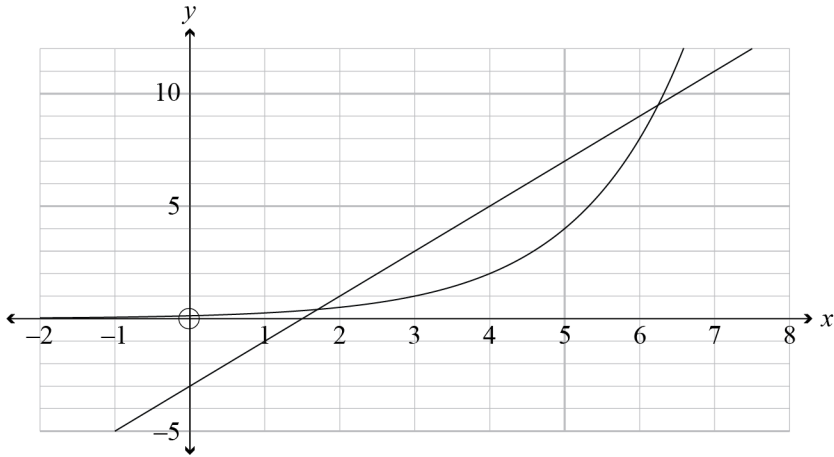


Q	Evidence	Achievement	Achievement with Merit	Achievement with Excellence
TWO (a)	$y = -(x-3)^2 + 6$ OR $y = -x^2 + 6x - 3$ OR $y = -(x-1)(x-5) + 2$ Allow other equivalent solutions.	<ul style="list-style-type: none"> <li>Correct equation.</li> </ul>		
(b)	Draw the graph $y = 2^{x-3}$ Draw the graph $y = 2x - 3$ Intersection at $x = 1.7$ and $x = 6.25$ Allow margin of error in the accuracy, consistent with the graph drawing. Allow an error of $\pm 0.2$ .	<ul style="list-style-type: none"> <li>Drawing the graph of <math>y = 2^{x-3}</math>  OR  Consistent solutions from minor error.  OR  Only one solution provided.  OR  CAO</li> </ul>	<ul style="list-style-type: none"> <li>Two values of <math>x</math> found from the intersection of the two graphs.</li> <li>Graph must show both intersection points</li> </ul>	
(c)(i)	Graph drawn, as discrete points, for $0 < x \leq 8$ Allow for minor error in graph	<ul style="list-style-type: none"> <li>Graph drawn, as a continuous graph.  Do not penalise negative values included.</li> </ul>	<ul style="list-style-type: none"> <li>Graph drawn as discrete points for <math>0 \leq x \leq 8</math>  OR <math>0 &lt; x \leq 8</math></li> </ul>	E7 / T1 Correct discrete graph drawn for $0 \leq x \leq 8$ OR $0 < x \leq 8$ AND Correct justified equation, but not with an appropriate domain.
(ii)	Equation found, with some justification, e.g. second difference of +4 indicated in the table. $F = 2n^2 + 4n$ OR $F = 2n(n+2)$ OR $F = 2(n+1)^2 - 2$ AND Domain for $1 \leq x \leq 8$ (with integer points) Also allow for $x > 0$ or equivalent Allow equation given in terms of $y$ and $x$ .	<ul style="list-style-type: none"> <li>Recognition that the equation is a quadratic with the coefficient of <math>x^2</math> as 2.  OR  Table of first and second differences shown and indication that the equation is a quadratic.  OR  C.A.O.</li> </ul>	<ul style="list-style-type: none"> <li>Correct equation, with some valid justification.</li> </ul>	Correct justified equation, but not with an appropriate domain. OR Correct justified equation, with appropriate domain BUT with a continuous graph E8 / T2 Correct discrete graph drawn for $0 \leq x \leq 8$ OR $0 < x \leq 8$ AND Correct justified equation, with appropriate domain
(iii)	Exponential equation of $F = 4^{n-1}$ . OR $F = 0.25 \times 4^n$ Allow other equivalent solutions. Allow C.A.O.	<ul style="list-style-type: none"> <li>Equation identified as an exponential, with base of 4.</li> </ul>	<ul style="list-style-type: none"> <li>Correct equation of <math>F = 4^{n-1}</math>.</li> </ul>	

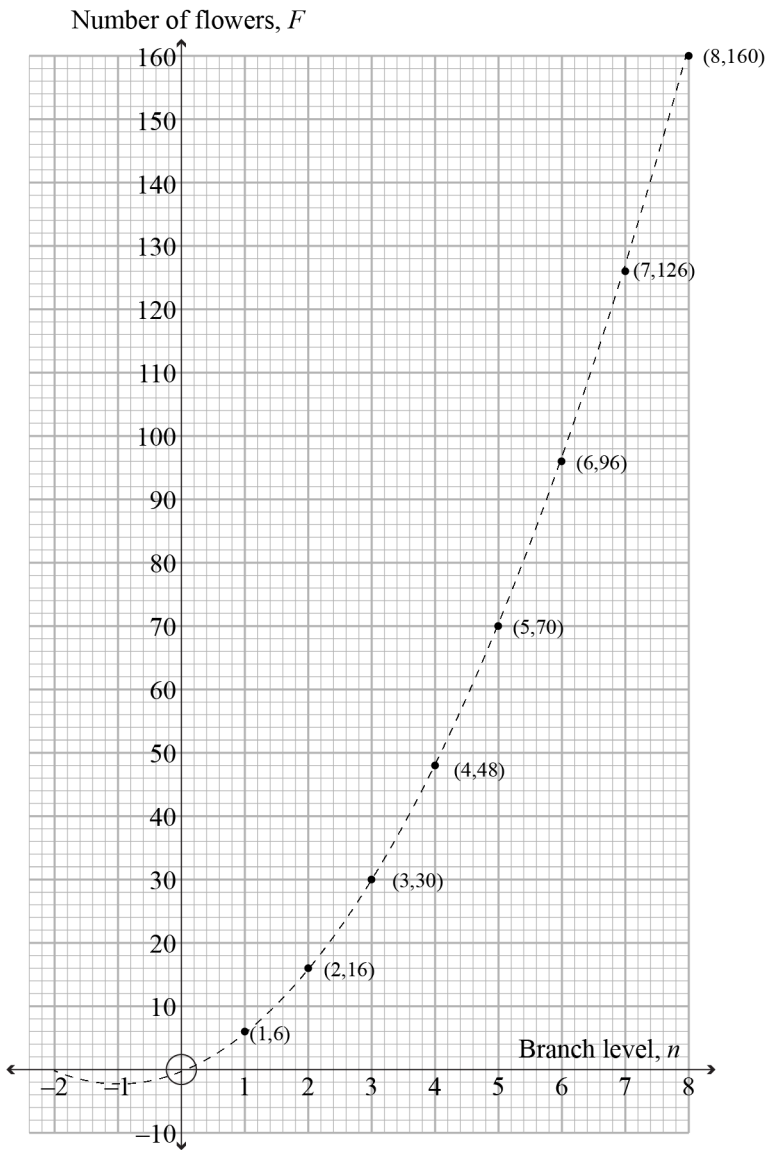
(iv)	<p>Sample comments regarding the false claim are:</p> <ul style="list-style-type: none"> <li>• No, it is only one set of results from one tree, so cannot be generalised to claim that this will always occur.</li> <li>• No, these results are from only one tree in one particular place, so cannot be generalised to all trees in NZ.</li> <li>• No, different growing conditions in different locations will lead to different results and hence a different formula.</li> <li>• No, the results are only for up to branch 8 flowers. Other-sized trees may not follow the same pattern.</li> <li>• No, trees may be diseased, which would affect the number of flowers on the branches.</li> </ul> <p>Allow other valid reasons.</p>	<ul style="list-style-type: none"> <li>• Recognising that the claim is false.</li> </ul> <p>AND</p> <p>With ONE valid comment.</p>		
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NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE question attempted towards solution.	1u	2u	3u	2r	3r	1t	2t

Question Two (b)



Question Two (c)(i)



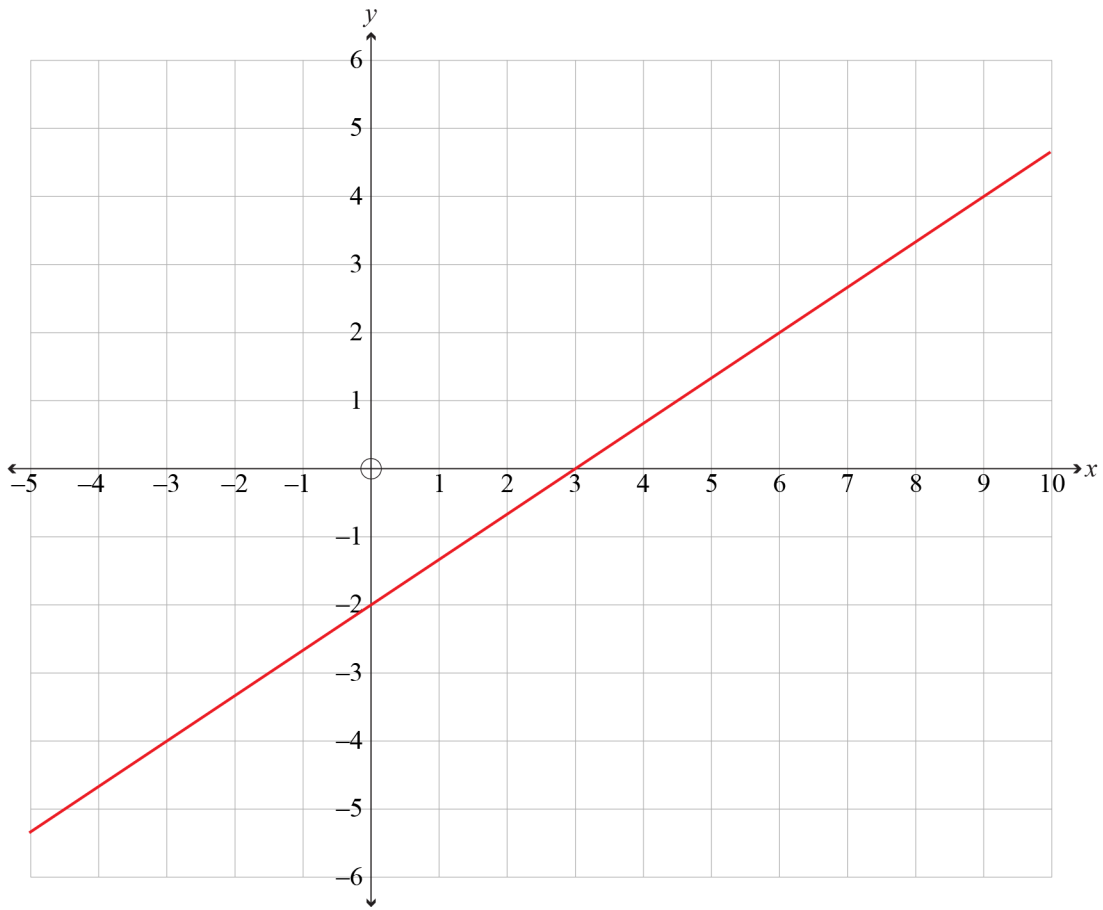
Q	Evidence	Achievement	Achievement with Merit	Achievement with Excellence
THREE (a)(i)	Correct equation of $y = 2^{x+2} - 4$ OR equivalent.	<ul style="list-style-type: none"> <li>Equation identified as an exponential, with base of 2.</li> </ul>	<ul style="list-style-type: none"> <li>Correct equation.</li> </ul>	
(ii)	Straight line graph drawn, showing $x$ -axis intercept at (3,0) AND $y$ -axis intercept at (0,-2).	<ul style="list-style-type: none"> <li>Straight line with only one axis-intercept correct.</li> </ul>	<ul style="list-style-type: none"> <li>Accurate graph drawn.</li> </ul>	
(b)(i)	Correct equation of $y = 1000 \times 1.2^x$ OR $S = 1000 \times 1.2^t$	<ul style="list-style-type: none"> <li>Included 1000 in the exponential equation.</li> <li>OR</li> <li>Recognised that the base is 1.2.</li> </ul>	<ul style="list-style-type: none"> <li>Correct equation.</li> </ul>	



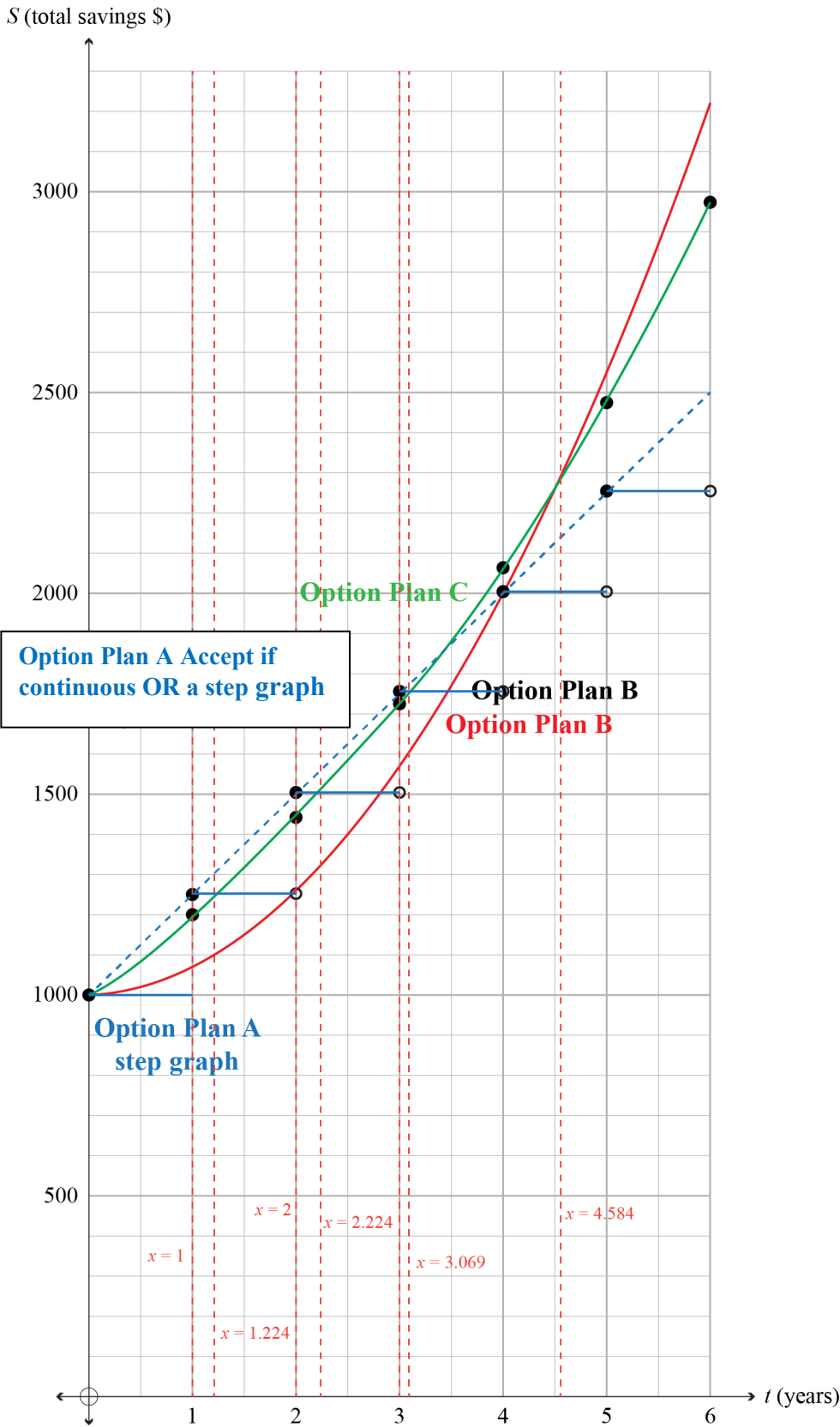
(ii)	Graph drawn for Savings Plan A.	<ul style="list-style-type: none"> <li>Continuous graph drawn of <math>y = 250x + 1000</math></li> </ul>		<p><b>E7/T1</b> Savings Plan A drawn as a continuous graph OR a correct step-graph</p> <p>AND</p> <p>Equation for savings plan C</p> <p>AND</p> <p>Savings Plan B drawn accurately.</p> <p>AND</p> <p>At least two valid comparisons made before and after the intercept point of <math>t = 4.6</math> years.</p> <p><b>E8 / T2</b> As for E7</p> <p>AND</p> <p>At least three valid comparisons of the options made both before and after the intercept point of <math>t = 4.6</math> years.</p>
	Graph drawn for Savings Plan B Parabola continuous graph.	<ul style="list-style-type: none"> <li>Parabola graph drawn, showing (0,1000) and at least THREE other values drawn, but lacking accuracy.</li> </ul>	<ul style="list-style-type: none"> <li>Parabola graph drawn correctly</li> </ul>	
(iii)	<p>Table of values for Savings Plan A and Savings Plan B produced.</p> <p>Valid comparisons made between the various Option Plans, including the evidence of dates, at least.</p> <p>Examples of possible comparison comments are:</p> <ul style="list-style-type: none"> <li>Generally, Savings Plan C will be the best if <math>t &lt; 4.6</math> years (approximately).</li> <li>Generally, Savings Plan B will be the best if <math>t &gt; 4.6</math> years (approximately).</li> <li>As the years increase, so Savings Plan B will become better and better compared to the other savings plans.</li> <li>Occasionally, but for only short time periods, Savings Plan A is the best, just after 1 year and just after 2 years.</li> <li>Savings Plan A is generally the weakest choice.</li> </ul> <p>Other non-trivial valid comparisons acceptable.</p>	<ul style="list-style-type: none"> <li>Table for Savings Plan A OR Savings Plan B correct</li> <li>AND</li> <li>ONE valid, non-trivial comparison made.</li> </ul>	<ul style="list-style-type: none"> <li>Savings Plan A AND Savings Plan B correct in Table</li> <li>AND</li> <li>TWO valid non-trivial comparisons made before and after <math>t = 4.6</math> years. (Can use whole number of years)</li> </ul>	

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE question attempted towards solution.	1u	2u	3u	2r	3r	1t	2t

Question Three (a)(ii)



Question Three (b)(ii)



Question Three (b)(iii)

End of Year ( <i>t</i> )	Total Savings Option Plan A	Total Savings Option Plan B	Total Savings Option Plan C
0	1000	1000	1000
1	1250	1070	1200
2	1500	1260	1440
3	1750	1570	1728
4	2000	2000	2073.60
5	2250	2550	2488.32
6	2500	3220	2985.98

**Cut Scores**

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
0 – 6	7 – 13	14 – 18	19 – 24

**Assessment Schedule – 2023****Mathematics and Statistics: Investigate relationships between tables, equations and graphs (91028)****Evidence**

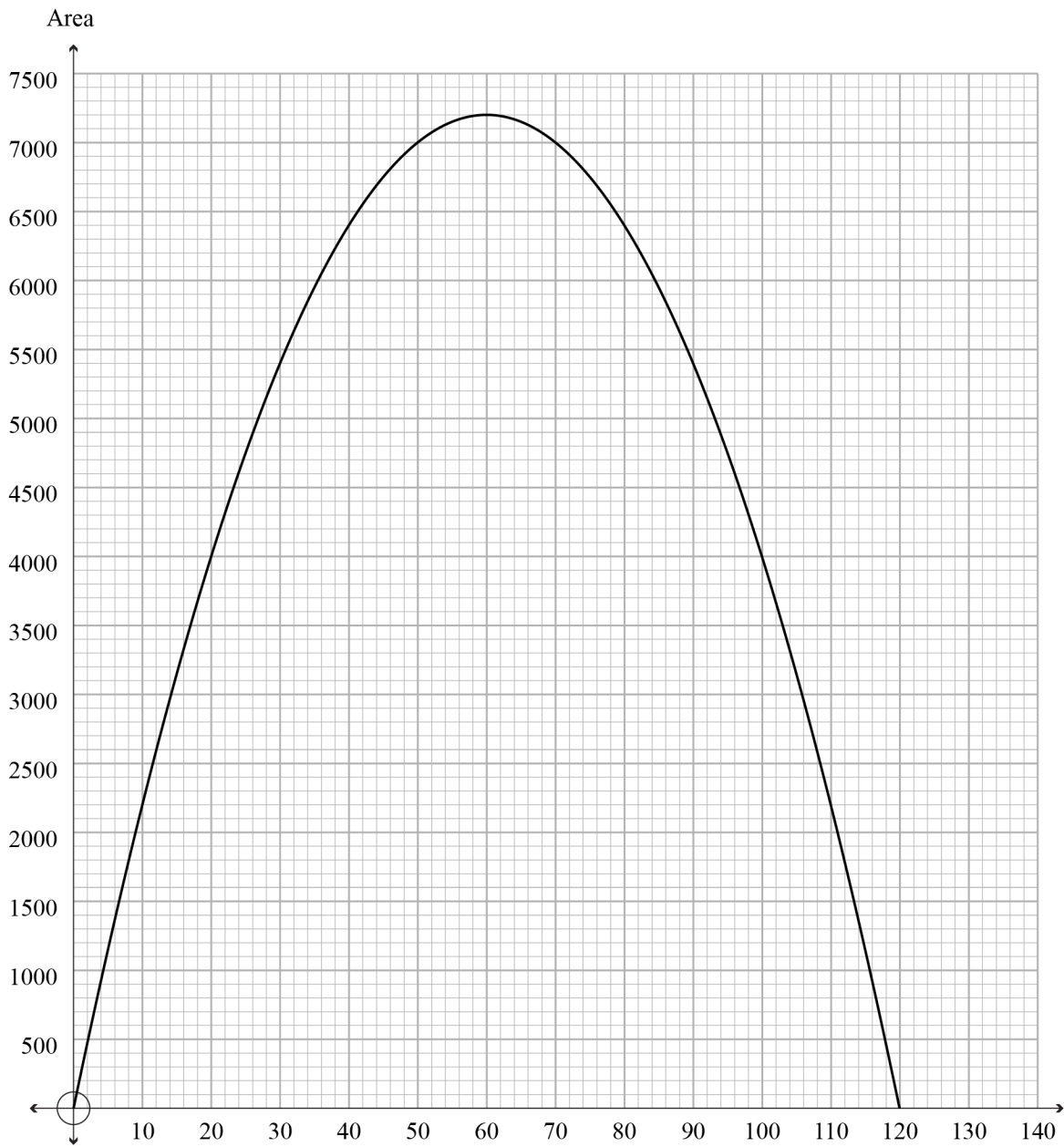
Q	Evidence	Achievement	Achievement with Merit	Achievement with Excellence
ONE (a)	$y = \frac{-5}{2}x + 15$ <p>Allow alternative forms. Allow C.A.O.</p>	<ul style="list-style-type: none"> <li>Correct equation.</li> </ul>		
(ii)	$y = \frac{5}{2}x + 15 + 10$ $y = \frac{5}{2}x + 25$ <p>Allow alternative forms. Allow C.A.O.</p>	<ul style="list-style-type: none"> <li>Correct equation after one of the transformations. OR Both transformations correct from wrong equation in (a)(i).</li> </ul>	<ul style="list-style-type: none"> <li>Correct equation after both of the transformations.</li> </ul>	
(b)(i)	$H = k(x - 60)^2 + 36$ <p><math>x = 0, y = 45</math> gives</p> $k = \frac{9}{3600} = \frac{1}{400} = 0.0025$ <p>i.e.</p> $H = \frac{1}{400}(x - 60)^2 + 36$ <p>OR Alternative formats:</p> $H = \frac{1}{400}x(x - 120) + 45$ <p>OR</p> $H = \frac{1}{400}x^2 - \frac{3}{10}x + 45$	<ul style="list-style-type: none"> <li>Equation given but with no <math>k</math>-value considered. OR Attempt made to find the value of <math>k</math> in a correct set up of the equation. OR C.A.O.</li> </ul>	<ul style="list-style-type: none"> <li>Correct equation for <math>H</math>, including full and clear working.</li> </ul>	
(ii)	<p>Possible changes are :</p> <p>The whole graph could be shifted downwards. This would represent shifting downwards where the chain fixes onto the post. This would be shown in the equation by reducing the size of the constant at the end.</p> <p>e.g. <math>H = \frac{1}{400}(x - 60)^2 + 20</math></p> <p>This would lower the chain totally by 16 cm. OR other examples where the chain is lowered.</p>	<ul style="list-style-type: none"> <li>Valid suggestion of how the equation should be changed. OR Example of equation of new design.</li> </ul>	<ul style="list-style-type: none"> <li>Valid suggestion of how the equation should be changed with an example equation AND Description of the minimum point of the chain fence, in context .</li> </ul>	

(c)	<p>Total perimeter (3 sides).  <math>2x + y = 240</math>  <math>y = 240 - 2x</math>                  Area = <math>x(240 - 2x)</math></p> <p>Allow other versions of this equation, e.g. <math>y = -2(x - 60)^2 + 7200</math>.                  (Allow any correct equation which starts with  <math>x + 2y = 240</math>)                  e.g. <math>y = -\frac{1}{2}(x - 120)^2 + 7200</math></p> <p>Table produced of the relationship between the two sides of the grassed space and their area with at least 5 correct values.</p> <p>Graph produced relating length of one side and area.</p> <p>Evidence of the use of tables, equations, and graphs to model the area of the grassed space as the lengths of the sides change.</p> <p>Sample comments:</p> <ul style="list-style-type: none"> <li>• Maximum area is 7200 m<sup>2</sup>.</li> <li>• Maximum area is when <math>x = 60</math> m and <math>y = 120</math> m.</li> <li>• Graph and area size is symmetrical.</li> <li>• Minimum area is 0 cm<sup>2</sup> (theoretically).</li> <li>• Rate of increase of the area changes for different <math>x</math>-values.</li> <li>• The graph will be a continuous one, as all different <math>x</math>-values are possible, if measurements are taken accurately.</li> <li>• In reality, some of the <math>x</math>-values close to 0 or close to 120 are likely to be inappropriate for the council to design their grassed area with these dimensions.</li> </ul>	<ul style="list-style-type: none"> <li>• Forming equation for area in terms of only one variable.</li> <li>OR</li> <li>Table only with one non-trivial comment.</li> <li>OR</li> <li>Graph only with ONE non-trivial comment.</li> <li>OR</li> <li>Finding maximum area only.</li> <li>OR</li> <li>Table and graph drawn with no comments.</li> </ul>	<ul style="list-style-type: none"> <li>• Evidence of only two aspects of tables, equations, and graphs.</li> <li>AND</li> <li>TWO non-trivial comments.</li> </ul>	<p><b>E7 / T1</b></p> <p>Evidence of table of values.                  AND                  Graph drawn.                  AND                  Formula for area provided.                  BUT                  Only maximum area discussed.                  OR                  As evidence for E8 but graph is discrete or of poor quality.</p> <p><b>E8 / T2</b></p> <p>Evidence of table of values.                  AND                  Graph drawn.                  AND                  Formula for area provided.                  AND                  At least three valid non-trivial comments.</p>
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N0	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE question attempted towards solution.	1u	2u	3u	2r	3r	1t	2t

### Question One

$x$	$y = 240 - 2x$	$A = x(240 - 2x)$
0	240	0
20	200	4000
40	160	6400
60	120	7200
80	80	6400
100	40	4000
120	0	0



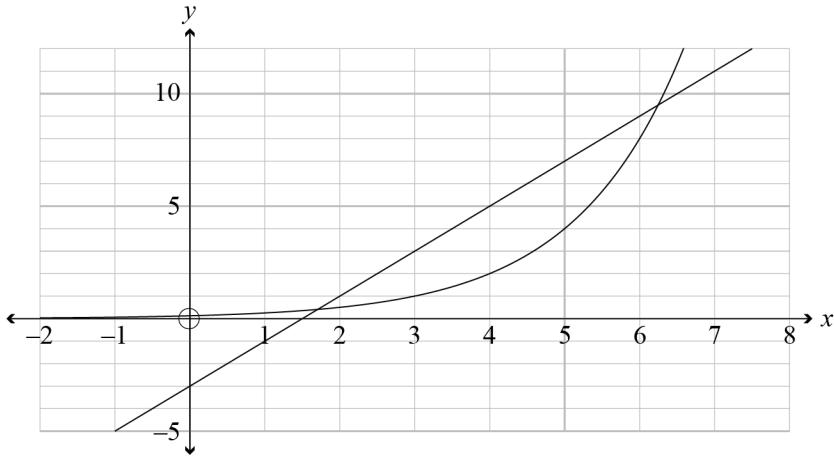
Q	Evidence	Achievement	Achievement with Merit	Achievement with Excellence
TWO (a)	$y = -(x-3)^2 + 6$ OR $y = -x^2 + 6x - 3$ OR $y = -(x-1)(x-5) + 2$ Allow other equivalent solutions.	<ul style="list-style-type: none"> <li>Correct equation.</li> </ul>		
(b)	Draw the graph $y = 2^{x-3}$ Draw the graph $y = 2x - 3$ Intersection at $x = 1.7$ and $x = 6.25$ Allow margin of error in the accuracy, consistent with the graph drawing. Allow an error of $\pm 0.2$ .	<ul style="list-style-type: none"> <li>Drawing the graph of <math>y = 2^{x-3}</math>  OR  Consistent solutions from minor error.  OR  Only one solution provided.  OR  CAO</li> </ul>	<ul style="list-style-type: none"> <li>Two values of <math>x</math> found from the intersection of the two graphs.</li> <li>Graph must show both intersection points</li> </ul>	
(c)(i)	Graph drawn, as discrete points, for $0 < x \leq 8$ Allow for minor error in graph	<ul style="list-style-type: none"> <li>Graph drawn, as a continuous graph.  Do not penalise negative values included.</li> </ul>	<ul style="list-style-type: none"> <li>Graph drawn as discrete points for <math>0 \leq x \leq 8</math>  OR <math>0 &lt; x \leq 8</math></li> </ul>	E7 / T1 Correct discrete graph drawn for $0 \leq x \leq 8$ OR $0 < x \leq 8$ AND Correct justified equation, but not with an appropriate domain.
(ii)	Equation found, with some justification, e.g. second difference of +4 indicated in the table. $F = 2n^2 + 4n$ OR $F = 2n(n+2)$ OR $F = 2(n+1)^2 - 2$ AND Domain for $1 \leq x \leq 8$ (with integer points) Also allow for $x > 0$ or equivalent Allow equation given in terms of $y$ and $x$ .	<ul style="list-style-type: none"> <li>Recognition that the equation is a quadratic with the coefficient of <math>x^2</math> as 2.  OR  Table of first and second differences shown and indication that the equation is a quadratic.  OR  C.A.O.</li> </ul>	<ul style="list-style-type: none"> <li>Correct equation, with some valid justification.</li> </ul>	Correct justified equation, but not with an appropriate domain. OR Correct justified equation, with appropriate domain BUT with a continuous graph E8 / T2 Correct discrete graph drawn for $0 \leq x \leq 8$ OR $0 < x \leq 8$ AND Correct justified equation, with appropriate domain
(iii)	Exponential equation of $F = 4^{n-1}$ . OR $F = 0.25 \times 4^n$ Allow other equivalent solutions. Allow C.A.O.	<ul style="list-style-type: none"> <li>Equation identified as an exponential, with base of 4.</li> </ul>	<ul style="list-style-type: none"> <li>Correct equation of <math>F = 4^{n-1}</math>.</li> </ul>	



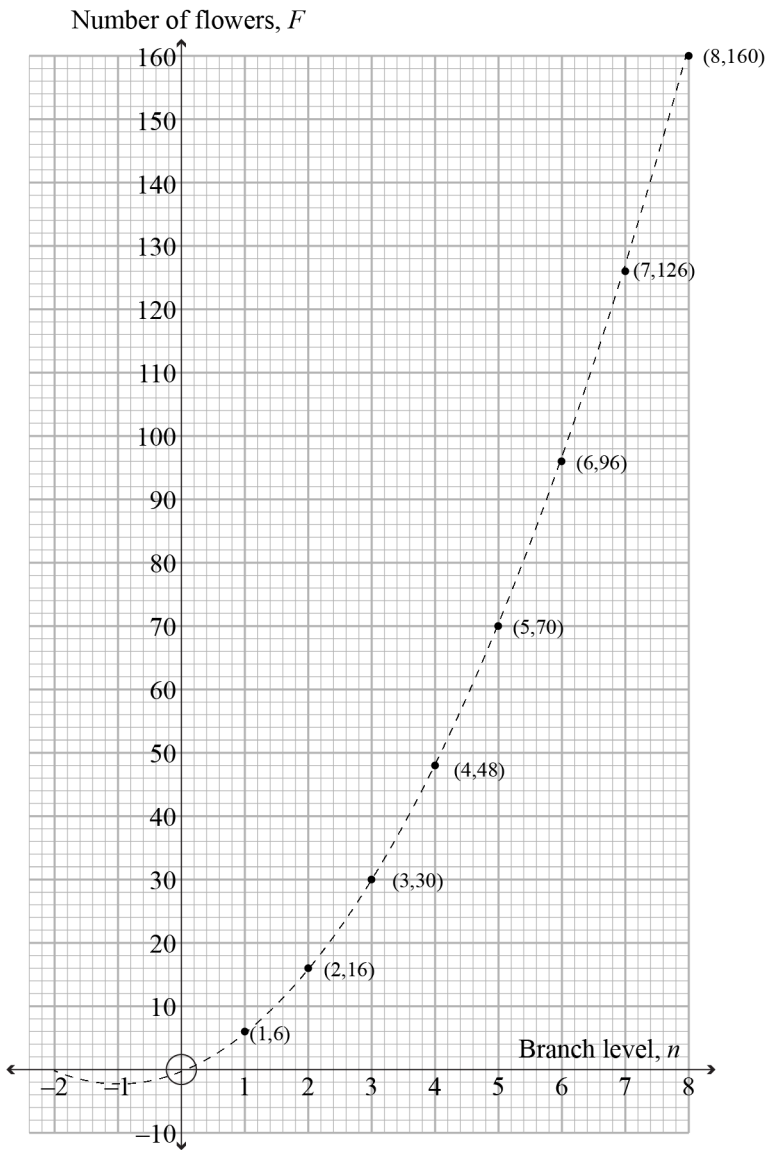
(iv)	<p>Sample comments regarding the false claim are:</p> <ul style="list-style-type: none"> <li>• No, it is only one set of results from one tree, so cannot be generalised to claim that this will always occur.</li> <li>• No, these results are from only one tree in one particular place, so cannot be generalised to all trees in NZ.</li> <li>• No, different growing conditions in different locations will lead to different results and hence a different formula.</li> <li>• No, the results are only for up to branch 8 flowers. Other-sized trees may not follow the same pattern.</li> <li>• No, trees may be diseased, which would affect the number of flowers on the branches.</li> </ul> <p>Allow other valid reasons.</p>	<ul style="list-style-type: none"> <li>• Recognising that the claim is false.</li> </ul> <p>AND</p> <p>With ONE valid comment.</p>		
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NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE question attempted towards solution.	1u	2u	3u	2r	3r	1t	2t

Question Two (b)



Question Two (c)(i)

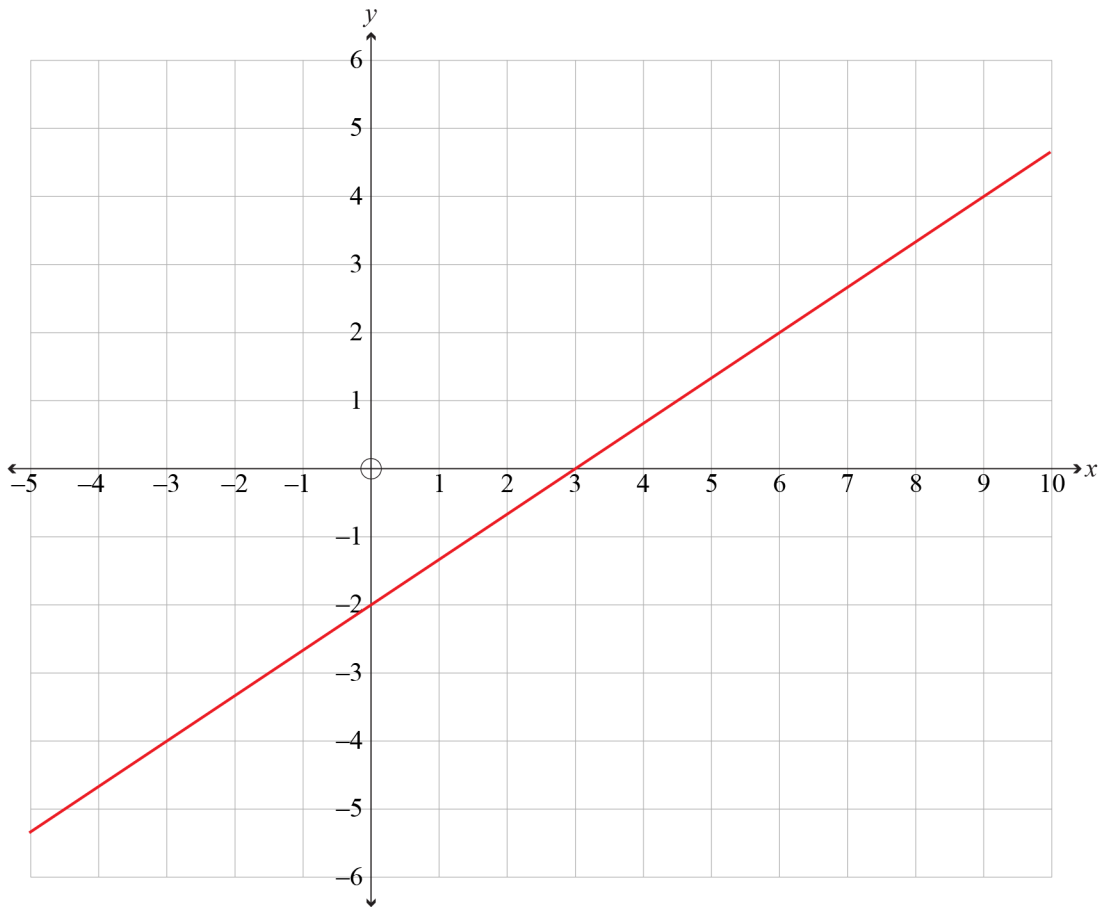


Q	Evidence	Achievement	Achievement with Merit	Achievement with Excellence
THREE (a)(i)	Correct equation of $y = 2^{x+2} - 4$ OR equivalent.	<ul style="list-style-type: none"> <li>Equation identified as an exponential, with base of 2.</li> </ul>	<ul style="list-style-type: none"> <li>Correct equation.</li> </ul>	
(ii)	Straight line graph drawn, showing $x$ -axis intercept at (3,0) AND $y$ -axis intercept at (0,-2).	<ul style="list-style-type: none"> <li>Straight line with only one axis-intercept correct.</li> </ul>	<ul style="list-style-type: none"> <li>Accurate graph drawn.</li> </ul>	
(b)(i)	Correct equation of $y = 1000 \times 1.2^x$ OR $S = 1000 \times 1.2^t$	<ul style="list-style-type: none"> <li>Included 1000 in the exponential equation.</li> <li>OR</li> <li>Recognised that the base is 1.2.</li> </ul>	<ul style="list-style-type: none"> <li>Correct equation.</li> </ul>	

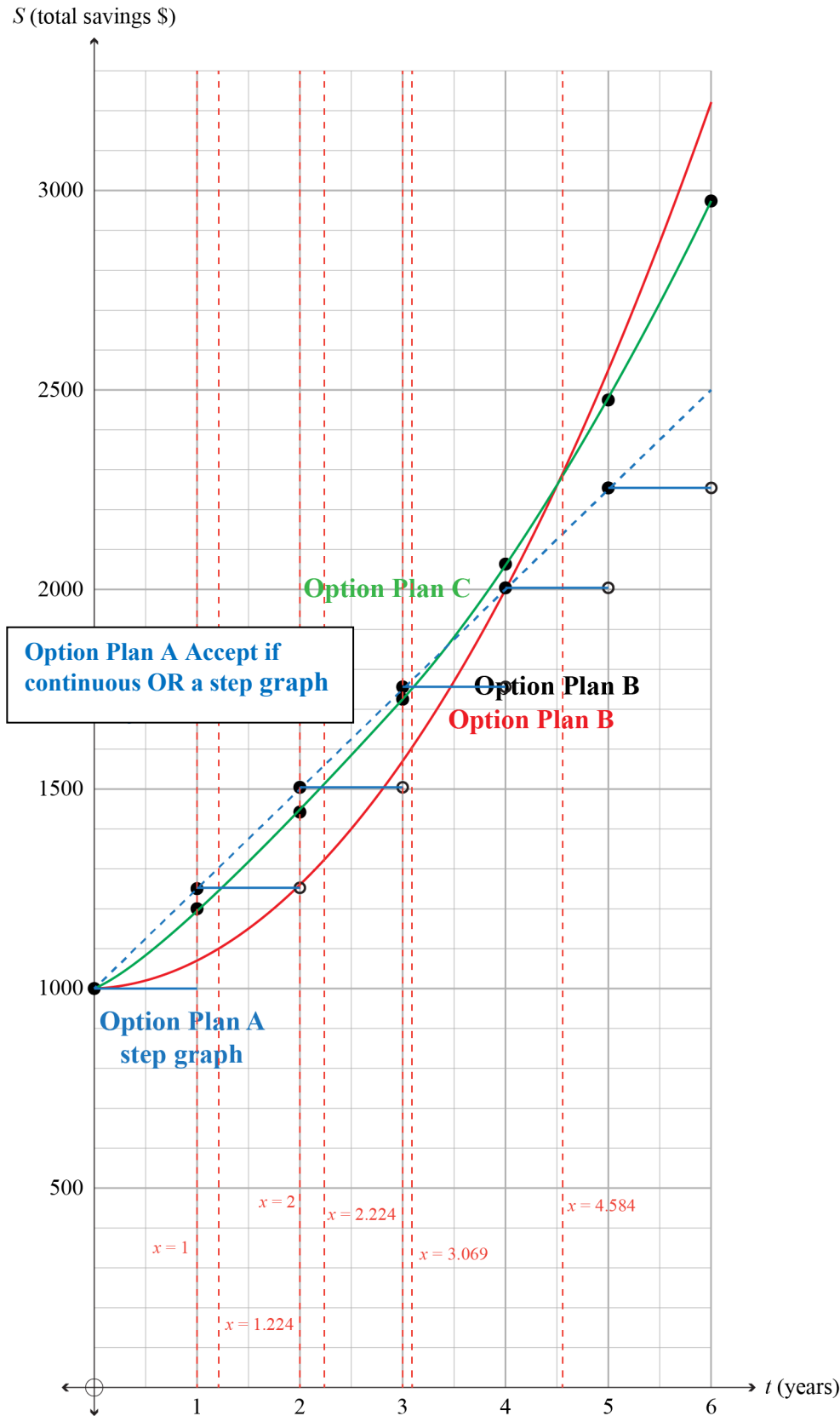
(ii)	Graph drawn for Savings Plan A.	<ul style="list-style-type: none"> <li>Continuous graph drawn of <math>y = 250x + 1000</math></li> </ul>		<p><b>E7/T1</b> Savings Plan A drawn as a continuous graph OR a correct step-graph</p> <p>AND</p> <p>Equation for savings plan C</p> <p>AND</p> <p>Savings Plan B drawn accurately.</p> <p>AND</p> <p>At least two valid comparisons made before and after the intercept point of <math>t = 4.6</math> years.</p> <p><b>E8 / T2</b> As for E7</p> <p>AND</p> <p>At least three valid comparisons of the options made both before and after the intercept point of <math>t = 4.6</math> years.</p>
	Graph drawn for Savings Plan B Parabola continuous graph.	<ul style="list-style-type: none"> <li>Parabola graph drawn, showing (0,1000) and at least THREE other values drawn, but lacking accuracy.</li> </ul>	<ul style="list-style-type: none"> <li>Parabola graph drawn correctly</li> </ul>	
(iii)	<p>Table of values for Savings Plan A and Savings Plan B produced.</p> <p>Valid comparisons made between the various Option Plans, including the evidence of dates, at least.</p> <p>Examples of possible comparison comments are:</p> <ul style="list-style-type: none"> <li>Generally, Savings Plan C will be the best if <math>t &lt; 4.6</math> years (approximately).</li> <li>Generally, Savings Plan B will be the best if <math>t &gt; 4.6</math> years (approximately).</li> <li>As the years increase, so Savings Plan B will become better and better compared to the other savings plans.</li> <li>Occasionally, but for only short time periods, Savings Plan A is the best, just after 1 year and just after 2 years.</li> <li>Savings Plan A is generally the weakest choice.</li> </ul> <p>Other non-trivial valid comparisons acceptable.</p>	<ul style="list-style-type: none"> <li>Table for Savings Plan A OR Savings Plan B correct</li> <li>AND</li> <li>ONE valid, non-trivial comparison made.</li> </ul>	<ul style="list-style-type: none"> <li>Savings Plan A AND Savings Plan B correct in Table</li> <li>AND</li> <li>TWO valid non-trivial comparisons made before and after <math>t = 4.6</math> years. (Can use whole number of years)</li> </ul>	

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE question attempted towards solution.	1u	2u	3u	2r	3r	1t	2t

Question Three (a)(ii)



Question Three (b)(ii)



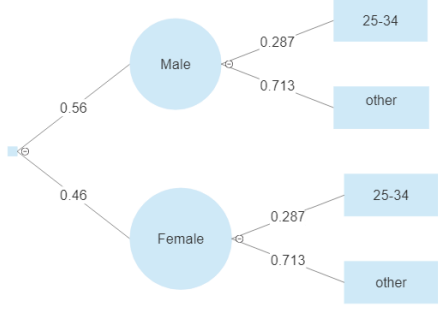
Question Three (b)(iii)

End of Year ( <i>t</i> )	Total Savings Option Plan A	Total Savings Option Plan B	Total Savings Option Plan C
0	1000	1000	1000
1	1250	1070	1200
2	1500	1260	1440
3	1750	1570	1728
4	2000	2000	2073.60
5	2250	2550	2488.32
6	2500	3220	2985.98

**Cut Scores**

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
0 – 6	7 – 13	14 – 18	19 – 24

**Assessment Schedule – 2023****Mathematics and Statistics: Demonstrate understanding of chance and data (91037)****Evidence**

Q	Evidence	Achievement	Achievement with Merit	Achievement with Excellence
ONE (a)(i)	$18.8\% + 10.9\% = 29.7\%$ or $0.297$	<ul style="list-style-type: none"> <li>Correct answer.</li> </ul>		
(ii)	<p>Prob (female and 25–34)  <math>= 0.46 \times 0.287</math>  <math>= 0.132</math>  OR</p> 	<ul style="list-style-type: none"> <li>C.A.O.</li> <li>OR</li> <li>Partial tree diagram.</li> </ul>	<ul style="list-style-type: none"> <li>Correct probability, with working.</li> </ul>	
(iii)	<ul style="list-style-type: none"> <li>Expected value = <math>63 \times 0.188 = 11.84</math>  The expected number of aged 55+ users is 12 people.  (Allow 11 people; Allow 11 or 12 people; Allow about 12 people.)  .Niko's percent is 12.6%</li> <li>Because the sample size is quite small out of the huge number of Spotify users, Niko should expect to see quite a large variation.</li> <li>The result of 8 Spotify users is only 3 Spotify users less than the expected value of 11 Spotify users.</li> <li>Niko's claim is correct, as this small difference between his result and the actual expected value is an acceptable variation within his relatively small sample size.</li> <li>Candidate queries whether the sample chosen by Niko is actually a representative random sample as the survey members have been selected from only Nico's family and work colleagues.  This selection may cause bias in the results.</li> </ul>	<ul style="list-style-type: none"> <li>Stating sample size is quite small.</li> <li>OR</li> <li>The sample may have possible bias.</li> </ul>	<ul style="list-style-type: none"> <li>Calculated expected value.</li> <li>OR</li> <li>Calculating probabilities</li> </ul>	<p><b>T1 / E7</b>  As for r</p> <p>AND  Discussed the large sampling variability based on small sample size.</p> <p>OR  Discussed issues regarding the sample selection and possible bias.</p>



(b)(i)	Feb 2017, when there were approximately 185-195 million users.	<ul style="list-style-type: none"> <li>• Correct answer, with some evidence.</li> </ul>		
(ii)	<p><b>Trend</b></p> <p>The long-term trend has increased from approximately 210 million users in May 2016 to just over 280 million users in Aug 2020.</p> <p><b>Unusual</b></p> <ul style="list-style-type: none"> <li>• There is a large spike in late 2017, jumping from 190 million users in Feb to all time high of 320 million users in Dec 2017.</li> <li>• A second noticeable spike was in Dec 2019, reaching around 287 million users.</li> <li>• Do not accept any reference to non-existent repeating patterns.</li> <li>• Irregular scale.</li> <li>• No obvious seasonality.</li> </ul>	<ul style="list-style-type: none"> <li>• Any one sensible feature identified.</li> <li>Accept omission of justification.</li> </ul>	<ul style="list-style-type: none"> <li>• Any two sensible features identified, with attempt to justify.</li> </ul>	<p><b>T1 / E7</b></p> <p>Two valid features with clear numerical evidence, justification, units.</p> <p><b>T2 / E8</b></p> <p>Gaining T1 AND identifying a grade r – quality misleading feature on the graph (part (iii)).</p> <p>OR</p> <p>Three valid features with clear numerical evidence, justification, units.</p>
(iii)	<p>The timeline is not on a linear scale (regular interval).</p> <p>The vertical scale starts at 175, but scales should be starting at 0.</p> <p>Because the vertical scale starts at 175, this would have the effect of exaggerating the actual rises in the data.</p> <p>Accept other non-trivial valid comments.</p>	<ul style="list-style-type: none"> <li>• Identifying a non-trivial valid comment relating to the graph being misleading.</li> </ul>	<ul style="list-style-type: none"> <li>• Identifying two non-trivial valid comments relating to the graph being misleading.</li> </ul>	

<b>NØ</b>	<b>N1</b>	<b>N2</b>	<b>A3</b>	<b>A4</b>	<b>M5</b>	<b>M6</b>	<b>E7</b>	<b>E8</b>
No response or no relevant evidence.	One question part attempted.	1u	2u	3u	1r	2r	1t	2t

Q	Evidence	Achievement	Achievement with Merit	Achievement with Excellence
TWO (a)(i)	$\frac{17}{68} \times \frac{17}{68} = \frac{1}{16} = 0.0625$ OR $\frac{17}{68} \times \frac{16}{67} = \frac{4}{67} = 0.0597$ OR $0.25 \times 0.25 = 0.0625$	<ul style="list-style-type: none"> <li>Recognised the need to use <math>\frac{17}{68}</math> or 0.25.</li> </ul>	<ul style="list-style-type: none"> <li>Correct probability. (Allow sampling with or without replacement.)</li> </ul>	
(ii)	<p><b>Centre</b> The median of pop music tempo is 123 bpm, which is higher than the classical music tempo of 97 bpm</p> <p><b>Shift / Overlap</b> The middle 50% box for the pop music tempo is further up on the scale than the classical music tempo (must state values). There is some overlap between the two middle 50% boxes (must state values).</p> <p><b>Shape</b> The distribution of pop music tempo is almost symmetrical, whereas the classical music tempo is slightly right skewed.</p> <p><b>Spread</b> The IQR of classical music is 57 bpm compared to 45 bpm for pop music. OR Range classical 142, pop 136</p> <p><b>Clusters</b> Identifying classical at 70-90</p> <p><b>Unusual point (outlier)</b> 199 (or 198) for classical</p>	<ul style="list-style-type: none"> <li>ONE significant feature compared.</li> </ul>	<ul style="list-style-type: none"> <li>TWO different significant features compared, with some numerical evidence included.</li> </ul>	<ul style="list-style-type: none"> <li>THREE different significant features compared, including appropriate relevant numerical evidence.</li> </ul>
(iii)	<p>Thom's claim is false because pop music tempo tends to be faster / higher than classical music tempo on Spotify, because the median tempo of classical music (97 bpm) is lower than the LQ tempo (102 bpm) of pop music, AND including numerical justification.</p> <p>I am confident in my conclusion because the sample size is big enough for me to use <math>\frac{1}{2}</math> and <math>\frac{3}{4}</math> rule.</p> <p>OR</p> <p>Thom's claim is false, however, I am not very confident because the median is only just outside the pop music middle 50% box. Sampling variability suggests that a different sample may show both medians within each other's box, in which case Thom would be correct.</p> <p>(Do not accept that the sample size is not sufficiently large.)</p>	<ul style="list-style-type: none"> <li>Rejected claim with a valid attempt to justify.</li> </ul>	<ul style="list-style-type: none"> <li>Decision that the claim is false, including a correct conclusion, with reasoning based on classical median outside classical box OR DBM/OVS</li> </ul>	<ul style="list-style-type: none"> <li>Response as in Merit, including supporting numerical justification. AND Confidence level is clearly discussed.</li> </ul>

(b)(i)	$\frac{1-570}{1000} = \frac{430}{1000} = \frac{43}{100} = 0.43$	<ul style="list-style-type: none"> <li>• Correct answer.</li> </ul>		
(ii)	<p>Because the <b>sample size</b> of the graph is 1000 is large enough to allow an estimate for the centre of graph, I would expect to see a <b>similar</b> distribution.</p>	<ul style="list-style-type: none"> <li>• Similar distribution, with ONE distribution feature.</li> <li>OR</li> <li>New sample is likely to have 4 slow, 110 medium, 60 fast, 26 very fast pop songs.</li> <li>OR</li> <li>Sample size discussed.</li> </ul>	<ul style="list-style-type: none"> <li>• Similar distribution, with examples AND Sample size discussed</li> </ul>	

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response or no relevant evidence.	One question part attempted.	1u	2u	3u	1r	2r	1t	2t

Q	Evidence	Achievement	Achievement with Merit	Achievement with Excellence
THREE (a)(i)	<p>Danceability score value that is approximately 0.6.</p> <p>Allow the response given as a range, e.g. score would be between 0.3 to 0.7.</p> <p>Allow indicated on the graph.</p> <p>Allow evidence of averaging the 3 or 4 data points for the score of 0.1.</p> <p>I am not confident in this result because there are only 3 results for the score of 0.1 and they have a big variation.</p>	<ul style="list-style-type: none"> <li>Valid answer.</li> </ul>	<ul style="list-style-type: none"> <li>Valid answer for the expected “Danceability” score.</li> </ul> <p>AND</p> <p>Comment expressing a lack of confidence in the prediction with valid reasoning.</p>	
(ii)	<p>There appears to be a positive linear relationship. as “Happy music” score increases then so does the “Danceability” score.</p> <p>The relationship is weak, because the points are scattered away from the line of the best fit.</p> <p>Accept there is no relationship.</p>	<ul style="list-style-type: none"> <li>One statement made for the relationship, with some valid justification.</li> </ul>	<ul style="list-style-type: none"> <li>TWO statements made for the relationship, in context, with valid justification.</li> </ul>	
(iii)	<p><b>Overall</b>, as the relationship is weak, I would not be very confident in my predictions from these results.</p> <p><b>But</b>, I would be more confident to predict a danceability score for music with “Happy music” score higher than 0.75.</p> <p>Because the points are much <b>closer to the line of best fit</b> between 0.75 and 1 (showing a stronger relationship) than below 0.4.</p> <p><b>But</b>, I would be reluctant and very unconfident to predict a danceability score for music with “Happy music” score less than 0.4.</p> <p>Because most of the points are <b>scattered away from the line</b>. This section is showing the weakest relationship.</p>		<ul style="list-style-type: none"> <li>Correct conclusion, with some correct justification.</li> </ul> <p>E.g. The relationship is weaker between 0 and 0.4.</p>	<ul style="list-style-type: none"> <li>Correct conclusion, made with clear comparison of at least two sections AND with supporting justification.</li> </ul>

(b)(i)	$\frac{92}{500} = \frac{23}{125} = 0.184 = 18.4\%$	<ul style="list-style-type: none"> <li>• Correct probability.</li> </ul>		
(ii)	<p>Prob(less than 2 hours, given a free subscription)</p> $= \frac{40}{200} = 0.2 = 20\%$ <p>Prob(less than 2 hours, given a premium subscription)</p> $= \frac{21}{300} = 0.07 = 7\%$ <p>Must have valid numerical reasoning, e.g. because <math>0.2 &gt; 0.07</math> or <math>20\% &gt; 7\%</math></p> <p>It is more likely that free subscribers will use less than 2 hours per week compared to the premium subscribers, because the proportion / probability / percentage of free subscribers is much higher than that of the premium subscribers.</p>	<ul style="list-style-type: none"> <li>• ONE conditional probability calculated.</li> </ul>	<ul style="list-style-type: none"> <li>• TWO conditional probabilities calculated.</li> </ul>	<ul style="list-style-type: none"> <li>• Both probabilities correct AND compared AND fully justified with conclusion made.</li> </ul>

<b>N0</b>	<b>N1</b>	<b>N2</b>	<b>A3</b>	<b>A4</b>	<b>M5</b>	<b>M6</b>	<b>E7</b>	<b>E8</b>
No response; no relevant evidence.	One partial solution	1 of u	2 of u	3 of u	1 of r	2 of r	t1	t2

**Cut Scores**

<b>Not Achieved</b>	<b>Achievement</b>	<b>Achievement with Merit</b>	<b>Achievement with Excellence</b>
0 – 7	8 – 14	15 – 19	20 – 24

<p>(c)</p>	$20x^2 + 20x = (2x + 5)^2$ $20x^2 + 20x = 4x^2 + 20x + 25$ $16x^2 = 25$ $x^2 = \frac{25}{16}$ $x = \pm \frac{5}{4}$ Accept $x = \pm \sqrt{\frac{25}{16}}$ OR Alternative method: $20x^2 + 20x = (2x + 5)^2$ $20x^2 + 20x = 4x^2 + 20x + 25$ $16x^2 - 25 = 0$ $(4x - 5)(4x + 5) = 0$ $x = \pm \frac{5}{4}$ Allow answer in any form.	<p>For award of u:</p> <p>ONE of:</p> <ul style="list-style-type: none"> <li>• expansion and simplification of RHS</li> <li>• consistently solves, giving both solutions</li> </ul> <hr/> <p>For award of r:</p> <ul style="list-style-type: none"> <li>• correctly solves for both solutions.</li> </ul>
<p>(d)</p>	<p>Let <math>F</math> be the first part of the journey and <math>S</math> be the second part of the journey.</p> $F + S = 1600$ $\Rightarrow S = 1600 - F$ $\frac{1}{3}F = \frac{1}{5}S$ $\Rightarrow 5F = 3S$ $5F = 3(1600 - F)$ $5F = 4800 - 3F$ $8F = 4800$ $F = 600$ <p>Then <math>S = 1600 - 600 = 1000</math> km</p> OR Alternative method: <p>Let <math>x</math> be the distance covered in the first part of the journey; then <math>1600 - x</math> is the distance remaining.</p> $\frac{x}{3} = \frac{1600 - x}{5}$ $5x = 3(1600 - x)$ $5x = 4800 - 3x$ $8x = 4800$ $x = 600$ <p>Remaining distance is <math>1600 - 600 = 1000</math> km</p> <p>Units not needed.</p> <p>Allow alternative algebraic methods.</p>	<p>For award of u:</p> <p>ONE of:</p> <ul style="list-style-type: none"> <li>• forms the equation <math>\frac{1}{3}F = \frac{1}{5}S</math> or <math>5F = 3S</math></li> <li>• consistent combining of their equations in one variable.</li> </ul> <hr/> <p>For award of r:</p> <p>ONE of:</p> <ul style="list-style-type: none"> <li>• combining of the equations in one variable</li> <li>• consistent distances found for both parts of the journey.</li> </ul> <hr/> <p>For award of t:</p> <ul style="list-style-type: none"> <li>• correct distances found for both parts of the journey.</li> </ul>

<p>(e)</p> $\begin{aligned} & \left[ (2n+3)^2 - 2(2n+3) + 5 \right] - \left[ (2n+1)^2 - 2(2n+1) + 5 \right] \\ &= \left[ 4n^2 + 12n + 9 - 4n - 6 + 5 \right] - \left[ 4n^2 + 4n + 1 - 4n - 2 + 5 \right] \\ &= \left[ 4n^2 + 8n + 8 \right] - \left[ 4n^2 + 4 \right] \\ &= 8n + 4 \quad \#1 \\ &= 4(2n+1) \end{aligned}$ <p>This expression has a factor of 4, it is divisible by 4.</p> <p>OR</p> <p>Alternative method:</p> <p>Assume <math>n</math> is odd – not required to be stated.</p> $\begin{aligned} & \left[ (n+2)^2 - 2(n+2) + 5 \right] - \left[ n^2 - 2n + 5 \right] \\ &= \left[ n^2 + 4n + 4 - 2n - 4 + 5 \right] - \left[ n^2 - 2n + 5 \right] \\ &= \left[ n^2 + 2n + 5 \right] - \left[ n^2 - 2n + 5 \right] \\ &= 4n \end{aligned}$ <p>This expression has a factor of 4, it is divisible by 4.</p> <p>OR</p> <p>Alternative method:</p> <p>Assume <math>n</math> is even – not required to be stated.</p> $\begin{aligned} & \left[ (n+3)^2 - 2(n+3) + 5 \right] - \left[ (n+1)^2 - 2(n+1) + 5 \right] \\ &= \left[ n^2 + 6n + 9 - 2n - 6 + 5 \right] - \left[ n^2 + 2n + 1 - 2n - 2 + 5 \right] \\ &= \left[ n^2 + 4n + 8 \right] - \left[ n^2 + 4 \right] \\ &= 4n + 4 \\ &= 4(n+1) \end{aligned}$ <p>This expression has a factor of 4, it is divisible by 4.</p> <p>Accept any order of the differences considered.</p> <p>Allow alternative algebraic methods.</p> <p>OR</p> <p>Allow other valid algebraic methods used by considering other consecutive odd terms, e.g. <math>2n - 1</math> and <math>2n + 1</math>.</p>	<p>For award of u:</p> <p>ONE of:</p> <ul style="list-style-type: none"> <li>• correct expression for the difference between two successive odd-numbered patterns</li> <li>• consistent expansion and simplification of one of the square shaped brackets</li> <li>• consistently simplifies to an expression without any brackets (equivalent to #1)</li> <li>• any valid numerical working (using one example) with a clear justification.</li> </ul>
	<p>For award of r:</p> <p>ONE of:</p> <ul style="list-style-type: none"> <li>• correctly simplifies to an expression without any brackets #1</li> <li>• consistently uses algebraic reasoning and explanation to justify clearly that the result is divisible by 4</li> <li>• Any valid numerical working (using two or more examples) with a clear justification.</li> </ul>
	<p>For award of t:</p> <ul style="list-style-type: none"> <li>• uses algebraic reasoning and explanation to justify clearly that the result is divisible by 4.</li> </ul>

Q	Expected Coverage	Grade (generated by correctly demonstrating the procedures listed in EN4) Requirements are for the student responses to be correct (ignoring numerical errors) unless the statement specifies consistent.
TWO (a)	$40x^2 + 11x - 2 = (5x + 2)(8x - 1)$ i.e. $y = 8x - 1$ Allow C.A.O.  Accept $y = \frac{40x^2 + 11x - 2}{5x + 2}$	For award of u: <ul style="list-style-type: none"> <li>stating that <math>y = 8x - 1</math> or <math>y = \frac{40x^2 + 11x - 2}{5x + 2}</math></li> </ul>
(b)	$(6x + 5)(4x - 1) = (2x + 1)(12x - 1)$ $24x^2 - 6x + 20x - 5 = 24x^2 - 2x + 12x - 1$ $24x^2 + 14x - 5 = 24x^2 + 10x - 1 \quad \#1$ $14x - 5 = 10x - 1$ $14x - 10x = -1 + 5$ $4x = 4$ $x = 1$	For award of u: <ul style="list-style-type: none"> <li>expansion and simplification of a pair of brackets (LHS or RHS), forming a quadratic expression with three terms #1</li> </ul> For award of r: <ul style="list-style-type: none"> <li>equation solved to find <math>x = 1</math>.</li> </ul>
(c)	$\frac{2(4x + 1) - 5(3x - 4)}{10} \geq 5$ $\frac{8x + 2 - 15x + 20}{10} \geq 5$ $\frac{-7x + 22}{10} \geq 5 \quad \#1$ $-7x + 22 \geq 50$ $-7x \geq 28$ $x \leq \frac{28}{-7}$ $x \leq -4$ <p>Accept <math>x \leq \frac{-28}{7}</math> or <math>x \leq -4</math> or <math>-4 \geq x</math> as the final answer.</p>	For award of u: ONE of: <ul style="list-style-type: none"> <li>correct arrangement for both numerator and denominator (does not need to be expanded or simplified). Accept <math>8x + 2 - 15x - 20</math> for the numerator</li> <li>consistent solution found (with either <math>\leq</math>, <math>\geq</math> or = sign).</li> </ul> For award of r: ONE of: <ul style="list-style-type: none"> <li>correct linear inequation at #1</li> <li>equation solved to find <math>x = -4</math></li> <li>inequation solved to find <math>x \geq -4</math></li> <li>consistently reverses inequality sign due to mult/div of a negative number.</li> </ul> For award of t: <ul style="list-style-type: none"> <li>inequation solved to find <math>x \leq -4</math>.</li> </ul>



<p>(d)</p>	$\frac{4w}{5} = \frac{v(w+3)}{4}$ $16w = 5v(w+3) \quad \#1$ $16w = 5vw + 15v$ $16w - 5vw = 15v \quad \#2$ $w(16 - 5v) = 15v \quad \#3$ $w = \frac{15v}{16 - 5v} \quad \#4$ <p>Accept other equivalent solutions.</p>	<p>For award of u: ONE of:</p> <ul style="list-style-type: none"> <li>• cross-multiply #1</li> <li>• consistently collecting terms involving <math>w</math> and terms not involving <math>w</math> on different sides of the equation #2</li> <li>• consistently factorising the pair of terms involving <math>w</math> #3</li> <li>• consistently rearranging by dividing by the bracket #4.</li> </ul>
		<p>For award of r: TWO of:</p> <ul style="list-style-type: none"> <li>• cross-multiply #1</li> <li>• consistently collecting terms involving <math>w</math> and terms not involving <math>w</math> on different sides of the equation #2</li> <li>• consistently factorising the pair of terms involving <math>w</math> #3</li> <li>• consistently rearranging by dividing by the bracket #4</li> </ul> <p>OR</p> <ul style="list-style-type: none"> <li>• correctly states <math>v</math> in terms of <math>w</math>.</li> </ul>
		<p>For award of t: • correct rearrangement.</p>
<p>(e)</p>	$(3x - 1)(3x + 2) - x(x + 1) = 8$ $9x^2 + 6x - 3x - 2 - x^2 - x - 8 = 0$ $8x^2 + 2x - 10 = 0$ $4x^2 + x - 5 = 0$ $(4x + 5)(x - 1) = 0$ <p>Either <math>4x + 5 = 0</math></p> $x = -\frac{5}{4} \quad \text{Ignore as not appropriate.}$ <p>OR</p> $x - 1 = 0$ $x = 1$ <p>Units not required.</p>	<p>For award of u: ONE of:</p> <ul style="list-style-type: none"> <li>• forming correct expression <math>(3x - 1)(3x + 2)</math> simplified to <math>9x^2 + 3x - 2</math></li> <li>• forming correct equation for the shaded area</li> <li>• consistent simplification to a quadratic equation in three terms.</li> </ul>
		<p>For award of r: ONE of:</p> <ul style="list-style-type: none"> <li>• simplification to a quadratic equation in three terms</li> <li>• consistent solving of their quadratic equation, with evidence of negative value disregarded</li> </ul>
		<p>For award of t: • correct positive solution found for the question, with evidence of negative value disregarded.</p>

Q	Expected Coverage	Grade (generated by correctly demonstrating the procedures listed in EN4) Requirements are for the student responses to be correct (ignoring numerical errors) unless the statement specifies consistent.
THREE (a)	Perimeter = $6(3x + 4) = 60$ $18x + 24 = 60$ $18x = 36$ $x = \frac{36}{18} = 2$ OR Alternative method: Perimeter = $6(3x + 4) = 60$ $3x + 4 = 10$ $3x = 6$ $x = \frac{6}{3} = 2$ Allow solution as an unsimplified fraction.	For award of u: <ul style="list-style-type: none"> <li>correct solution for the value of <math>x</math></li> </ul> Accept C.A.O.
(b)	$\frac{2x}{2x-3} = \frac{x+4}{x+2}$ $2x(x+2) = (x+4)(2x-3)$ $2x^2 + 4x = 2x^2 - 3x + 8x - 12$ $4x = 5x - 12$ $x = 12$ OR Alternative method: $\frac{2x(x+2) - (x+4)(2x-3)}{(2x-3)(x+2)} = 0$ $\frac{2x^2 + 4x - (2x^2 + 5x - 12)}{(2x-3)(x+2)} = 0$ $2x^2 + 4x - 2x^2 - 5x + 12 = 0$ $-x + 12 = 0$ $x = 12$	For award of u: ONE of: <ul style="list-style-type: none"> <li>correct arrangement for both numerator and denominator (does not need to be expanded or simplified). Accept <math>2x^2 + 4x - 2x^2 + 5x + 12</math> for the numerator</li> <li>consistent solution found.</li> </ul> For award of r: <ul style="list-style-type: none"> <li>correct value for <math>x</math> found.</li> </ul>

(c)	$\frac{1}{2} \times (2x + 8)(x + 2) = 24$ $(x + 4)(x + 2) = 24$ $x^2 + 6x + 8 = 24$ $x^2 + 6x - 16 = 0$ $(x + 8)(x - 2) = 0$ <p>Either <math>x = -8</math> Ignore as not appropriate Or <math>x = 2</math></p> <p>OR</p> <p>Alternative method:</p>	<p>For award of u:</p> <p>ONE of:</p> <ul style="list-style-type: none"> <li>• form and simplify a quadratic expression for the area of the triangle, with or without the 24 being used</li> <li>• consistent solution found, with evidence of the invalid value disregarded.</li> </ul>
	$\frac{1}{2} \times (2x + 8)(x + 2) = 24$ $(2x + 8)(x + 2) = 48$ $2x^2 + 4x + 8x + 16 - 48 = 0$ $2x^2 + 12x - 32 = 0$ $x^2 + 6x - 16 = 0$ $(x + 8)(x - 2) = 0$ <p>Either <math>x = -8</math> Ignore as not appropriate. Or <math>x = 2</math></p>	<p>For award of r:</p> <ul style="list-style-type: none"> <li>• correct solution found, with evidence of the invalid value disregarded.</li> </ul>

<p>(d)</p>	$(2^2)^{x-2} \times 2^{x+1} = (2^5)^x$ $2^{2x-4} \times 2^{x+1} = 2^{5x}$ $2^{2x-4+x+1} = 2^{5x} \quad \#1$ $2^{3x-3} = 2^{5x}$ $3x - 3 = 5x \quad \#2$ $-3 = 2x$ $x = -\frac{3}{2}$	<p>For award of u: ONE of:</p> <ul style="list-style-type: none"> <li>recognition of powers of 2 on both sides</li> <li>LHS or RHS correct at stage # 1.</li> </ul>
		<p>For award of r: ONE of :</p> <ul style="list-style-type: none"> <li>forming the linear equation # 2</li> <li>consistently forming an equation and solving for their <math>x</math>-value.</li> </ul>
		<p>For award of t: • correct solution found.</p>
<p>(e)</p>	<p>Let <math>T</math> be the number of \$20 notes. Let <math>F</math> be the number of \$50 notes.</p> $T + F = 40$ $20T + 50F = 1700$ $T + F = 40$ $2T + 5F = 170$ $2T + 2F = 80$ $2T + 5F = 170$ <p>Subtracting gives:</p> $3F = 90$ $F = 30 \text{ and } T = 10$ <p>Value of \$20 notes will be <math>10 \times \\$20 = \\$200</math> Value of \$50 notes will be <math>30 \times \\$50 = \\$1500</math></p> <p>or alternative method:</p> $20(40 - F) + 50F = 1700$ $F = 30$ $T = 10$ <p>Candidate could use <math>x</math> and <math>y</math> as the variables. Allow alternative algebraic methods.</p>	<p>For award of u: ONE of:</p> <ul style="list-style-type: none"> <li>forms both equations</li> <li>consistent combining of their equations into one variable.</li> </ul>
		<p>For award of r: ONE of:</p> <ul style="list-style-type: none"> <li>consistent number of either \$20 notes or \$50 notes</li> <li>correct value for either \$20 notes or \$50 notes</li> <li>correct combining of the equations into one variable</li> <li>consistent value for both \$20 notes and \$50 notes.</li> </ul>
		<p>For award of t: ONE of:</p> <ul style="list-style-type: none"> <li>correct number of both \$20 and \$50 notes</li> <li>correct value for both \$20 notes and \$50 notes.</li> </ul>